Austrian Journal of Statistics
2024, Volume 53, 39–51.
http://www.ajs.or.at/
doi:10.17713/ajs.v53i5.1898

Integrated LI-NSGA-II Approach for Solving the Non-linear Multi-objective Optimization Problem

Muskan Kapoor
Jaypee University
of Information Technology
Solan, HP

Bhupendra Kumar Pathak
Jaypee University
of Information Technology
Solan, HP

Rajiv Kumar
Jaypee University
of Information Technology
Solan HP

Abstract

Real-world engineering projects frequently involve complex, non-linear multi-objective optimization challenges. Traditional methods like PERT/CPM and basic heuristics often fail to provide optimal solutions in such scenarios. Multi-objective evolutionary algorithms, such as genetic algorithms and non-dominated sorting genetic algorithms, are more effective for identifying true Pareto solutions. Among these, the Non-dominated Sorting Genetic Algorithm-II (NSGA-II) is a well-known algorithm for solving multi-objective optimization problems. This study presents an integrated approach, called LI-NSGA-II, which combines Lagrange's Interpolation (LI) with NSGA-II to solve non-linear multi-objective optimization problems in real-world projects. In this LI-NSGA-II approach, LI is used to handle the non-linearity of competing objectives, whereas NSGA-II optimizes these objectives to find optimal solutions. The outcomes achieved through the LI-NSGA-II approach are ideal for real-time monitoring and control of non-linear multi-objective optimization problems.

Keywords: Lagrange's interpolation, multi-objective optimization, NSGA-II, multi-objective evolutionary algorithms, time-cost tradeoff problem.

1. Introduction

Multi-objective optimization (MOO) problems are a class of optimization problems in which many conflicting objectives must be optimized at the same time Deb, Sindhya, and Hakanen (2016). Unlike traditional single-objective optimization problems, which aim to discover a single optimal solution, MOO tries to uncover a set of solutions that represent the tradeoff between multiple objectives. These solutions are Pareto-optimal solutions because they are not dominated by any other possible solution in terms of all objectives Gunantara (2018). MOO can be complex because it must address competing objectives at the same time. To address this problem, researchers designed Multi Objective Evolutionary Algorithms (MOEAs), which are specialized algorithms that can effectively navigate the tradeoff space to find Pareto-optimal solutions Zitzler and Thiele (1999). The Time-Cost Tradeoff (TCT) problem, which is a type of multi-objective optimization, aims to minimize project completion time and costs simultaneously Hosseinpour, Roghanian, and Weber (2023). Genetic algorithm (GA) based techniques are well-suited for identifying these optimal solutions because there are numerous Pareto optimal solutions to TCT problems Fonseca and Fleming (1993). For resolving such problems, the evolutionary multi-objective optimization technique NSGA-II is widely established Dev (1999), Deb, Pratap, Agarwal, and Meyarivan (2002). De, Dunne, Ghosh, and Wells (1995) described numerous approaches for addressing the TCT problems. Using the application of computational intelligence paradigms, Srivastava, Pathak, and Srivastava (2010) addressed a range of TCT challenges that occur in engineering projects. Agdas, Warne, Osio-Norgaard, and Masters (2018) demonstrate that the meta-heuristic approach is efficient for large-scale constructing TCT problems and that GA can be utilized to solve vast benchmark networks of variables with high levels of accuracy consistently. Eirgash and Dede (2018) employed a multi-objective teaching-learning-based optimization (TLBO) method combined with the non-dominated sorting approach to optimize the projects and find the optimum set of time-cost alternatives. Dhawan, Sharma, and Trivedi (2020) provide a simulated annealing-based TCT model that makes use of the NSGA-II approach to enable the resolution of building activities with numerous alternatives and varying resource levels. A study by Sharma and Trivedi (2020) proposed an "NSGA-II based time-cost-quality tradeoff optimization model" where each project activity has distinct alternatives, each impacting time, cost, and quality, they also described a pairwise comparison-based analytical hierarchy process (AHP) to determine the relative importance of project activities and quality measures. Pathak, Srivastava, and Srivastava (2008) developed a method for project managers to perform sensitivity analysis of time-cost tradeoff profiles under various conditions. Lin, Lin, and Wei (2021) investigated the multi-objective evolutionary algorithms, NSGA-II, and vehicle multi-objective acceleration strategies that were employed to solve the multi-objective optimization problems. Tavassoli, Massah, Montazeri, Mirmozaffari, Jiang, and Chen (2021) created a tradeoff between reducing "preventative maintenance costs and minimizing the time needed to carry out this maintenance for a series-parallel system", using a modified version of NSGA-II. Albayrak (2020) emphasized the significant relationship between time and cost in a competitive environment, presenting a novel hybrid algorithm (NHA) for solving the TCT problem as a multi-objective problem. Likewise, several Multi-Objective Evolutionary Algorithms (MOEAs) have been suggested to find the solutions of multi-objective optimization problems. Hamta, Ehsanifar, and Sarikhani (2021) established a goal programming model for the "time-cost-quality tradeoff," where project managers aim to achieve goals with the lowest cost and highest quality. Chassiakos and Rempis (2019) assessed the performance of various evolutionary algorithms in order to address the time-cost tradeoff problems. To handle TCT problems, Akin, Polat, Turkoglu, and Damci (2021) proposed a model with an illustrated case study. Alavipour and Arditi (2019) created an integrated model for financing optimization and time-cost tradeoff analysis, using a hybrid GALP method combining genetic algorithms (GA) and linear programming (LP). Zhang, Xiao, Zhang, Lin, and Feng (2021) created an enhanced non-dominated sorting genetic algorithm-II with three modifications is offered to solve the proposed model in order to identify the roughly best Pareto solution set. Mrad, Al-gahtani, Hulchafo, Souayah, and Bamatraf (2019) demonstrated a model with very low computation time compared to other techniques, making it applicable even to large real-life projects. The Non-dominated Sorting Genetic Algorithm-II (NSGA-II) is a leading algorithm for solving multi-objective optimization problems. Recent developments have resulted in the first mathematical runtime guarantees for NSGA-II, although limited to synthetic benchmark problems Cerf, Doerr, Hebras, Kahane, and Wietheger (2023). Their results support the algorithm's outstanding empirical performance and show that mathematical analyses for NSGA-II can be applied to more complex combinatorial optimization problems Cerf *et al.* (2023). Stock price prediction is difficult due to the complexity of stock prices, which are influenced by non-linear, non-stationary, and high-dimensional factors Zeng, Cai, Liang, and Yuan (2023). To solve this, Zeng *et al.* (2023) presented I-NSGA-II-RF, an improved algorithm that combines random forest and a three-stage feature engineering procedure. This integration seeks to reduce computing complexity while improving forecast accuracy. Experimental results show that I-NSGA-II-RF outperforms other methods, including deep learning models, regarding accuracy, solution set size, and running time. Alimirzaloo, Biglari, Sadeghi, Keshtiban, and Sehat (2019) described a new way of optimizing the two-stage forging process of an airfoil blade. Alimirzaloo *et al.* (2019) used Lagrange interpolation to define design parameters, followed by finite element analysis and artificial neural network modeling to determine how input variables affect objectives. A multi-objective genetic algorithm, supported by fuzzy logic, is then utilized to determine the optimal parameters. Experimental results validate the technique, demonstrating significant improvements in flash volume and strain uniformity over non-optimized forging operations. Sharma, Khodadadi, Saha, Gharehchopogh, and Mirjalili (2023) presentd an enhanced Butterfly Optimization Algorithm (BOA) for multi-objective optimization, addressing its limitations and extending it into a more effective version called MONSBOA. Through testing on various benchmark and real-world problems, MONSBOA demonstrates superior performance in terms of Pareto front coverage and convergence speed compared to other algorithms.

Therefore, in most of the mentioned studies, many researchers used the evolutionary multi-objective optimization algorithms to solve linear multi-objective optimization problems like time-cost tradeoff problems. Furthermore, in non-linear MOO problems, Lagrange Interpolation with evolutionary multi-objective optimization algorithms has not been investigated by researchers.

This study addresses a real-world non-linear time-cost tradeoff problem, employing the LI-NSGA-II approach. In this approach, LI is utilized for estimating function values between known data points, effectively handling non-linearity. Even if the arguments are not evenly spaced, it is still possible to calculate the function's value using LI Séroul (2012), Das and Chakrabarty (2016), and Nguyen (2021). Subsequently, NSGA-II with genetic operators SBX and polynomial mutation is used for searching the optimal solutions.

Moreover, results from NSGA-II are compared with MOGA (Feng, Liu, and Burns (1997), Deb *et al.* (2002)) and demonstrated that the results obtained from NSGA-II are well-diversified and very close to the optimal front. Subsequently, the results obtained from LI-NSGA-II (non-linear) and NSGA-II (linear) are compared and shows that LI-NSGA-II approach is best suited for handling real-world non-linear multiobjective optimization problems.

2. Problem description

In this section, we examine a non-linear TCT problem, alternatively known as a multi-objective optimization problem Pathak and Srivastava (2015). The mathematical formulation of the non-linear TCT problem with two objectives is as follows:

Let an instance $\Omega = \{< t_i, c_i >| \; CT_i \leq t_i \leq NT_i, \; i = 1, 2, \ldots, n\}$ with t_i and c_i as the time and cost of the i^{th} activity respectively. n denotes the number of activities in the project network. NT_i and CT_i are the normal time and crash time of the i^{th} activity. Also, NC_i and CC_i represents the normal cost and the crash cost of the project activity respectively. Suppose the vector of decision variables is taken as $X = [x_1, x_2, ..., x_i, ..., x_n]$, where each i^{th} activity consist of k options.

Next, we have two conflicting objectives that require to be minimized, written as:

$$\text{minimize time } \; T = \sum_{i=1}^{n} t_i, \tag{1}$$

$$\text{minimize cost } \; C = \sum_{i=1}^{n} c_i, \tag{2}$$

where t_i and c_i can take any k^{th} option of i^{th} activity.

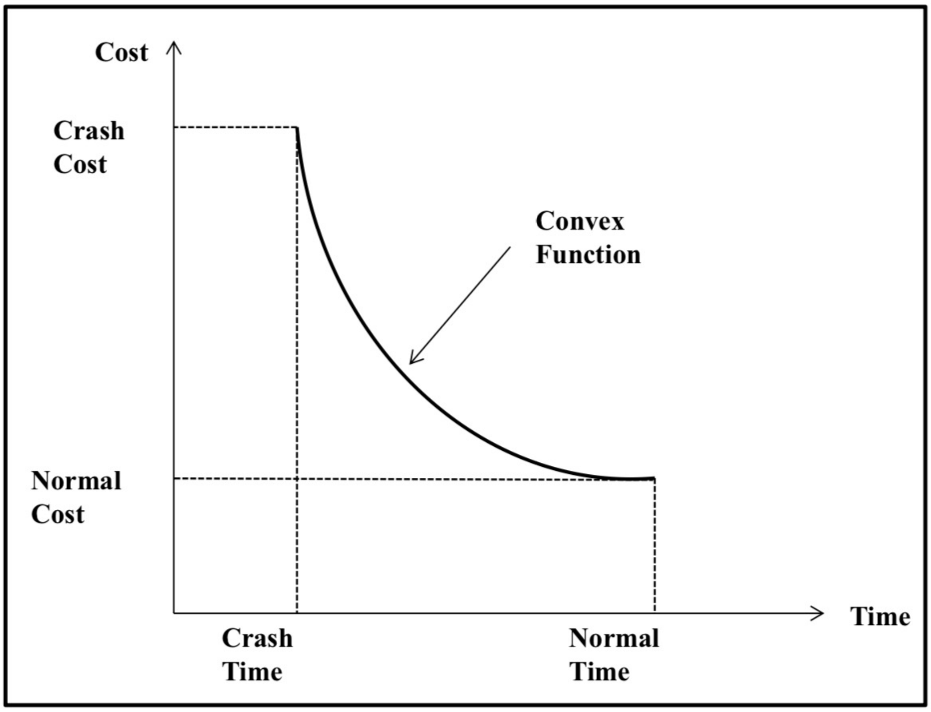

Figure 1: Non-linear time-cost tradeoff relationship for an activity

Figure 1 Pathak and Srivastava (2015) shows the non-linear model used in this paper to determine the relation between cost and time. The project duration T is determined by computing the maximum path time (critical path method).

3. Methodology of LI-NSGA-II approach

Incorporating LI into NSGA-II for solving non-linear multi-objective optimization problems involves the following steps:

3.1. Initialization of the population

The population initially consists of N_p solutions, each represented by a string $[t_1, t_2, ..., t_i, ..., t_n]$ in which $Ct_i \leq t_i \leq Nt_i$ for $i = 1, 2, ..., n$. This string denotes the project schedule's time. The associated cost (C) and duration (T) for each schedule are determined by calculating the maximum path time and aggregating activity costs. These solutions are termed as "parents."

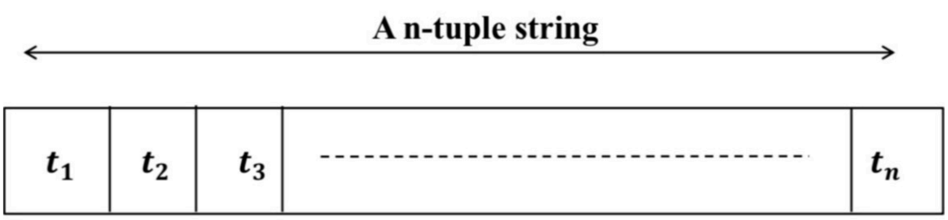

Figure 2: Time of the project schedule

3.2. LI for objective functions

This stage involves selecting relevant data points and interpolating between them to create a smooth approximation of the objective functions (time and cost). After initializing the random population with time as an input variable, the next step is to evaluate the cost using the following Lagrange Interpolation formula:

$$C = \frac{(t-t_2)(t-t_3)...(t-t_j)...(t-t_k)}{(t_1-t_2)(t_1-t_3)...(t_1-t_j)...(t_1-t_k)}c_1 + \frac{(t-t_1)(t-t_3)...(t-t_j)...(t-t_k)}{(t_2-t_1)(t_2-t_3)(t_2-t_j)...(t_2-t_k)}c_2 + ... + \\ + \frac{(t-t_1)(t-t_2)...(t-t_j)...(t-t_{k-1})}{(t_k-t_1)(t_k-t_2)...(t_k-t_j)...(t_k-t_{k-1})}c_k \quad (3)$$

For i-th activity, where t_j and c_j represent the j-th time and cost options, respectively, with $1 \leq j \leq k$ and $1 \leq i \leq n$. And t can take any random value between t_1 to t_k.

Therefore, LI-NSGA-II will generate a population of project activity completion times and use the Lagrange interpolation formula to estimate the cost for each activity corresponding to project completion time.

3.3. Non-dominated sort

The initial population (parents) is sorted using the non-domination algorithm Deb *et al.* (2002). In this approach, each solution is assigned a fitness (or rank) equal to its non-dominance level (1 is the best level, 2 is the next-best level, and so on). In order to discover the solutions of the primary non-dominated front in a population of size N_p, each solution will be compared to every other solution in the population to identify if it is dominated. At this point, every individual of the first non-dominated front has been located. The solution from the dominant front is temporarily discounted, and the method described above is applied once more to find the individuals of the second non-dominated front. This argument is true for finding third non-dominated front and so on.

3.4. Selection

Parents are chosen based on their capacity for reproduction in order to generate offspring. The original NSGA-II Deb *et al.* (2002) uses a binary tournament selection method based on the crowded-comparison operator in addition to selecting chromosomes to generate a mating pool. It is common to have this to be half of the population size. When the non-dominated sort is complete, the crowding distance is assigned. Because all the individuals in the population is selected based on their rank and crowding distance. Each individual in the population is assigned a crowding distance value. Since crowding distance is determined by front, comparing the crowding distance between two individuals in different front is meaningless.

3.5. Genetic operators

Crossover and mutation caused by the genetic operators are used to create offspring from parent chromosomes. There are a variety of crossover operators in evolutionary algorithms, however in this study; we focused primarily on simulated binary crossover and polynomial mutation Deb *et al.* (2002).

3.6. Intermediate population

Intermediate population is the combined population of parents and off-springs of the current generation. The population size is almost one and half times the initial population.

3.7. Non-dominated sort of intermediate population

The intermediate population is sorted again based on non-domination sort as explained in step 3.3.

3.8. Updating the population

Only the best solution is chosen once the intermediate population has been sorted based on its rank and crowding distance. Once the population has grown enough, each front gets filled in ascending order. Based on the individuals with the smallest crowding distance, the final front is included in the population.

The whole methodology of LI-NSGA-II is shown in Figure 3:

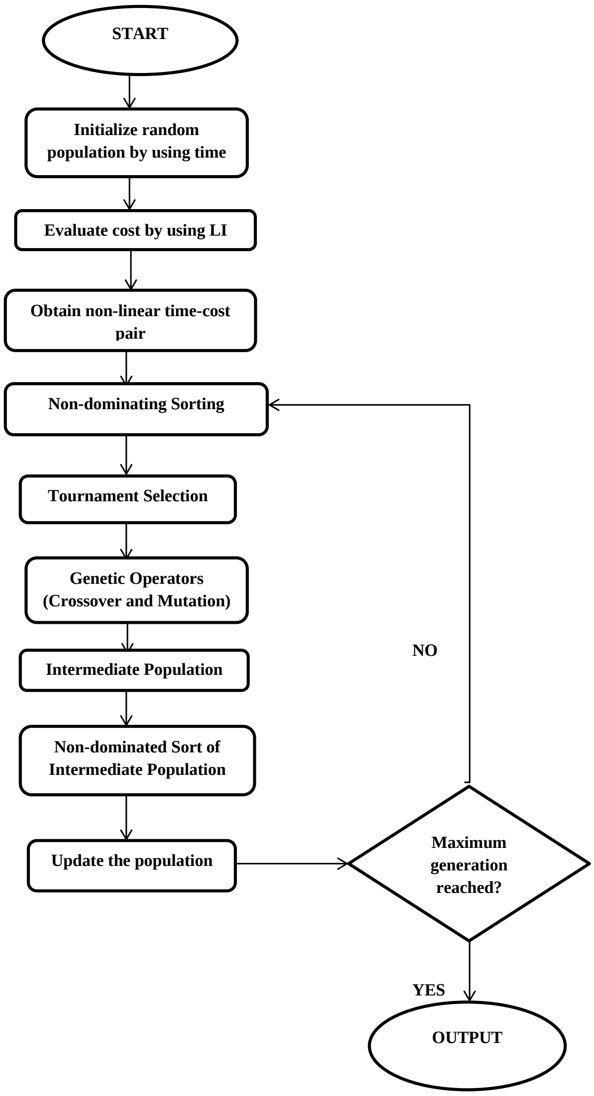

Figure 3: Flowchart of LI-NSGA-II

4. Results and discussion

The real-world project network depicted in Figure 4 Feng *et al.* (1997) is adapted as a test problem. Modifications are made to introduce a continuous and non-linear relation between activity time and cost. Subsequently, the proposed approach, LI-NSGA-II, is employed to address this problem. Furthermore, the characteristics of each activity are detailed out in Table 1.

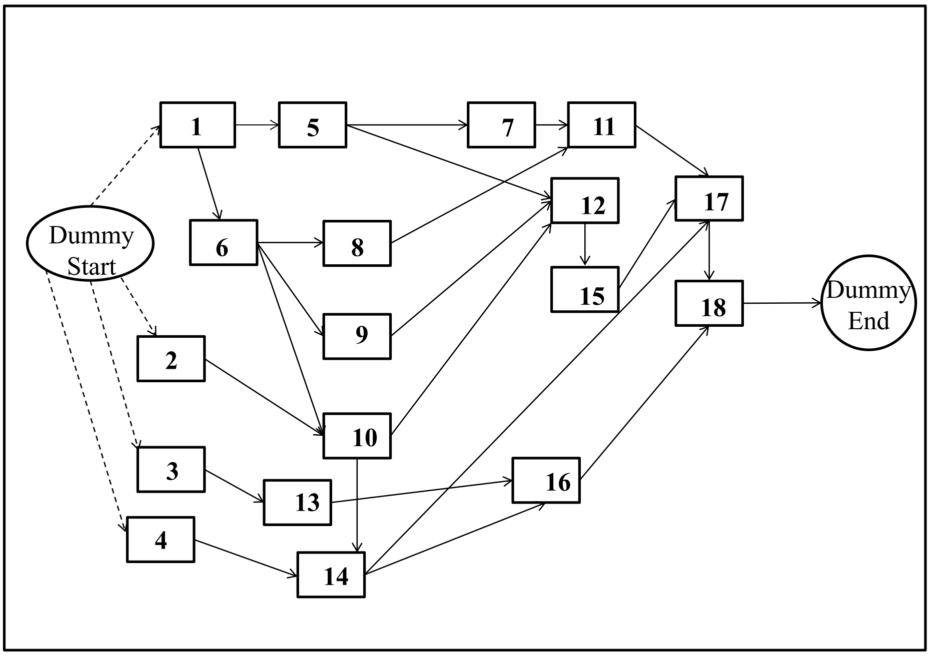

Figure 4: Network of test problem

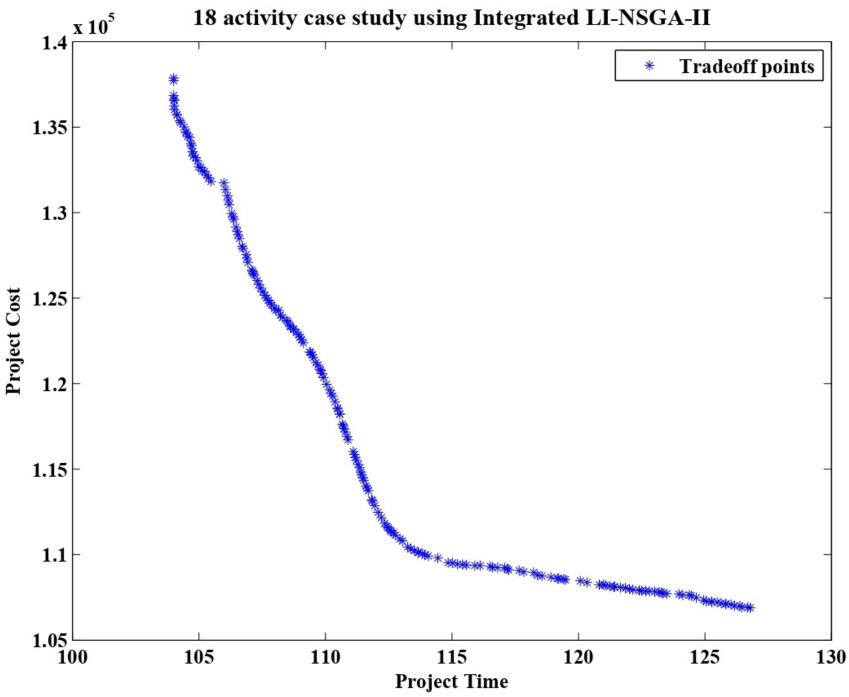

Figure 5: Tradeoff points using LI-NSGA-II

to solve this test problem using the proposed LI-NSGA-II approach, the initial population (n_p), mutation probability (p_m), and crossover probability are taken as 200, 0.05, and 0.9,

Table 1: Alternatives of test problem

ID	Time	Cost	ID	Time	Cost	ID	Time	Cost	ID	Time	Cost
1	14	2400	6	14	40000	11	12	450	16	20	3000
1	15	2150	6	16	39200	11	13	420	16	22	2000
1	16	1900	6	17	34500	11	14	370	16	24	1750
1	18	1750	6	18	32000	11	16	350	16	26	1685
1	21	1500	6	20	27700	11	17	330	16	28	1500
1	23	1340	6	22	20300	11	19	305	16	29	1385
1	24	1200	6	24	18000	11	20	300	16	30	1000
2	15	3000	7	9	30000	12	22	2000	17	14	4000
2	17	2630	7	11	27200	12	24	1750	17	16	3700
2	18	2400	7	13	26100	12	25	1690	17	17	3455
2	20	1800	7	14	25600	12	27	1525	17	18	3200
2	21	1720	7	15	24000	12	28	1500	17	21	2780
2	23	1500	7	17	22300	12	29	1200	17	23	2335
2	25	1000	7	18	22000	12	30	1000	17	24	1800
3	15	4500	8	14	220	13	14	4000	18	9	3000
3	17	4415	8	15	215	13	15	3795	18	10	2900
3	19	4220	8	16	200	13	16	3500	18	12	2790
3	22	4000	8	17	190	13	18	3200	18	14	2565
3	25	3730	8	21	167	13	21	2750	18	15	2400
3	30	3375	8	23	150	13	23	2155	18	16	2315
3	33	3200	8	24	120	13	24	1800	18	18	2200
4	12	45000	9	15	300	14	9	3000			
4	13	44300	9	16	240	14	10	2930			
4	15	38450	9	18	180	14	12	2825			
4	16	35000	9	21	150	14	14	2605			
4	18	33700	9	22	130	14	15	2400			
4	19	32400	9	24	110	14	17	2295			
4	20	30000	9	25	100	14	18	2200			
5	22	20000	10	15	450	15	10	6525			
5	24	17500	10	22	400	15	13	5990			
5	25	16400	10	23	390	15	14	4500			
5	26	15900	10	27	345	15	16	3500			
5	27	15700	10	28	330	15	17	3355			
5	28	15000	10	30	325	15	18	2600			
5	30	10000	10	33	320	15	20	1930			

respectively. Furthermore, if the tradeoff points remain constant for ten consecutive iterations, the search process is terminated. Alternatively, the search process ends after 1000 iterations, a suitable number determined through exhaustive experimentation. The detailed LI-NSGA-II results for this problem are shown in Figure 5, and the statistical analysis of these data is included in Table 2.

Table 2: Statistical analysis

Statistics	Min	Max	Mean	Median	Mode	Std	Range
Project Time	104	141.3	117.1	112.1	104	11.08	37.3
Project Cost	1.014e+05	1.321e+05	1.119e+05	1.083e+05	1.014e+05	9814	3.071e+04

5. Comparative results

The results obtained from NSGA-II are compared with MOGA (Feng *et al.* (1997), Pathak *et al.* (2008)) and illustrated in Figure 6. It is clear from Figure 6, that results obtained from NSGA-II are well-diversified and converges more effectively towards the optimal Pareto front. Therefore, NSGA-II is capable for solving real-world multi-objective optimization problems, as it is robust enough to maintain diversity among the solutions. Moreover, we conducted experiments assuming a non-linear relationship between the time and cost of each activity using LI-NSGA-II. Here, LI is used to capture the non-linear relationship of each activity, and NSGA-II is employed to obtain the optimal Pareto front, and these results are shown in Figure 7. Furthermore, the results achieved from LI-NSGA-II are then compared with those from NSGA-II (linear) and the comparative results are dipicted in Figure 7. The results obtained from LI-NSGA-II are particularly beneficial for addressing realistic, non-linear, multi-objective optimization problems.

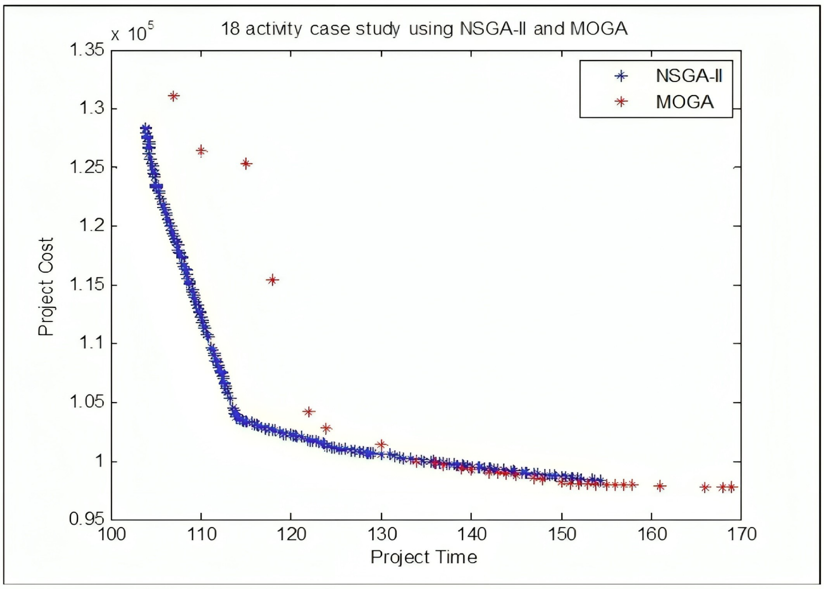

Figure 6: Tradeoff points by using NSGA-II and MOGA

6. Conclusion

The integrated LI-NSGA-II approach provided in this paper offers an innovative and effective solution for solving real-world nonlinear multi-objective optimization problems, with a particular emphasis on the time-cost tradeoff (TCT) problem in project management. This approach addresses the limitations of traditional methods in dealing with complex optimization challenges by combining Lagrange's Interpolation (LI) to capture the nonlinear relationship of each activity in a project network and the Non-dominated Sorting Genetic Algorithm-II (NSGA-II) to obtain optimal solutions. These results show that the LI-NSGA-II technique beats traditional methods and other evolutionary algorithms, such as MOGA and NSGA-II (linear), in terms of convergence to the optimal Pareto front. The LI-NSGA-II technique offers a wide range of solutions that are ideal for real-time monitoring and control of nonlinear multi-objective optimization problems. Furthermore, the application of LI enables a more precise representation of the nonlinear relationships between project activities, resulting in more realistic and practical solutions. Overall, the LI-NSGA-II approach described in this paper holds considerable promise for solving difficult optimization issues in a variety of domains, including project management, engineering, and finance. Future studies could investigate further improvements and the applicability of this approach to other difficult optimization problems.

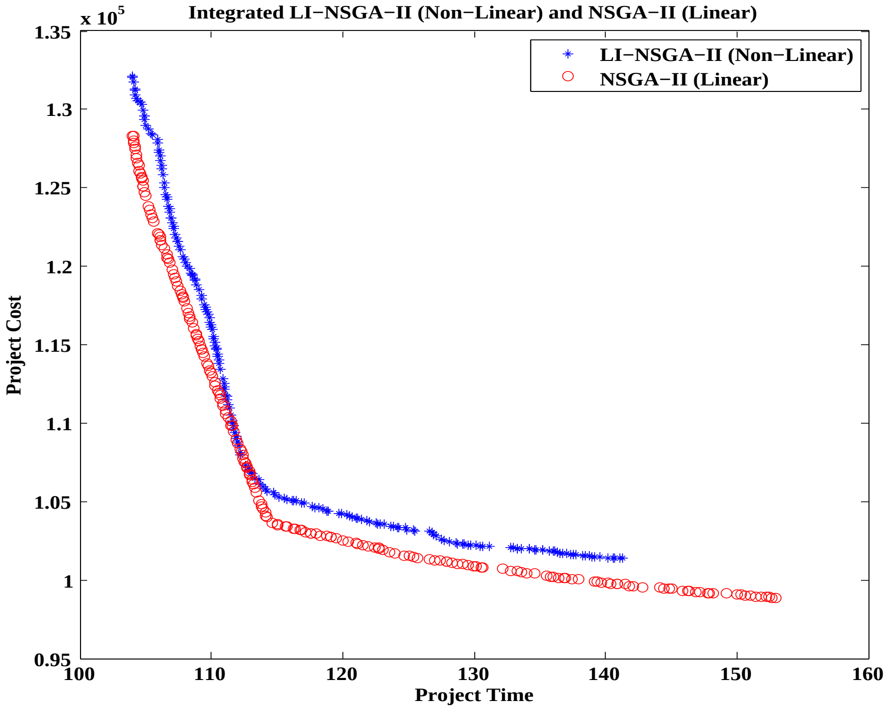

Figure 7: Tradeoff points by using LI-NSGA-II (Non-Linear) and NSGA-II (Linear)

References

Agdas D, Warne DJ, Osio-Norgaard J, Masters FJ (2018). "Utility of Genetic Algorithms for Solving Large-scale Construction Time-Cost Trade-off Problems." *Journal of Computing in Civil Engineering*, **32**(1). doi:10.1061/(asce)cp.1943-5487.0000718.

Akin FD, Polat G, Turkoglu H, Damci A (2021). "A Crashing-based Time-Cost Trade-off Model Considering Quality Cost and Contract Clauses." *International Journal of Construction Management*, **23**(6), 954–963. doi:10.1080/15623599.2021.1946899.

Alavipour SMR, Arditi D (2019). "Time-Cost Tradeoff Analysis with Minimized Project Financing Cost." *Automation in Construction*, **98**, 110–121. doi:10.1016/j.autcon.2018.09.009.

Albayrak G (2020). "Novel Hybrid Method in Time–Cost Trade-off for Resource-Constrained Construction Projects." *Iranian Journal of Science and Technology, Transactions of Civil Engineering*, **44**(4), 1295–1307. doi:10.1007/s40996-020-00437-2.

Alimirzaloo V, Biglari FR, Sadeghi MH, Keshtiban PM, Sehat HR (2019). "A Novel Method for Preform Die Design in Forging Process of an Airfoil Blade Based on Lagrange Interpolation and Meta-heuristic Algorithm." *The International Journal of Advanced Manufacturing Technology*, **102**, 4031–4045.

Cerf S, Doerr B, Hebras B, Kahane Y, Wietheger S (2023). "The First Proven Performance Guarantees for the Non-Dominated Sorting Genetic Algorithm II (NSGA-II) on a Combinatorial Optimization Problem." *arXiv preprint arXiv:2305.13459*.

Chassiakos AP, Rempis G (2019). "Evolutionary Algorithm Performance Evaluation in Project Time-Cost Optimization." *Journal of Soft Computing in Civil Engineering*, **3**, 16–29. doi:10.22115/SCCE.2019.155434.1091.

Das B, Chakrabarty D (2016). "Lagrange's Interpolation Formula: Representation of Numerical Data by a Polynomial Curve." *International Journal of Mathematics Trend and Technology*, **34**(2), 23–31. doi:10.14445/22315373/IJMTT-V34P514.

De P, Dunne EJ, Ghosh JB, Wells CE (1995). "The Discrete Time-Cost Tradeoff Problem Revisited." *European Journal of Operational Research*, **81**(2), 225–238. doi:10.1016/0377-2217(94)00187-H.

Deb K, Pratap A, Agarwal S, Meyarivan T (2002). "A Fast and Elitist Multiobjective Genetic Algorithm: NSGA-II." *IEEE Transactions on Evolutionary Computation*, **6**(2), 182–197.

Deb K, Sindhya K, Hakanen J (2016). "Multi-objective Optimization." In *Decision Sciences*, pp. 161–200. CRC Press.

Dev K (1999). "Multi-objective Genetic Algorithms: Problem Difficulties and Construction of Test Problems." *Evolutionary Computation*, **7**(3), 205–230.

Dhawan A, Sharma K, Trivedi MK (2020). "Simulated Annealing-Based Time—Cost Trade-Off Model for Construction." *Second International Conference on Sustainable and Innovative Solutions for Current Challenges in Engineering & Technology (ICSISCET- 2020) Madhav Institute of Technology & Science, Gwalior 18 - 19 December, 2020.* doi:10.1007/978-981-16-1220-6.

Eirgash MA, Dede T (2018). "A Multi-objective Improved Teaching Learning Based Optimization Algorithm for Time-Cost Trade-off Problems." *Journal of Construction Engineering, Management and Innovation*, **1**(3), 118–128. doi:10.31462/jcemi.2018.03118128.

Feng CW, Liu L, Burns SA (1997). "Using Genetic Algorithms to Solve Construction Time-Cost Trade-off Problems." *Journal of Computing in Civil Engineering*, **11**(3), 184–189. doi:10.1061/(asce)0887-3801(1997)11:3(184).

Fonseca CM, Fleming PJ (1993). "Genetic Algorithms for Multiobjective Optimization: Formulation Discussion and Generalization." In *Icga*, volume 93, pp. 416–423.

Gunantara N (2018). "A Review of Multi-objective Optimization: Methods and Its Applications." *Cogent Engineering*, **5**(1), 1502242.

Hamta N, Ehsanifar M, Sarikhani J (2021). "Presenting a Goal Programming Model in the Time-Cost-Quality Trade-off." *International Journal of Construction Management,*, **21**(1), 1–11. doi:10.1080/15623599.2018.1502930.

Hosseinpour M, Roghanian E, Weber G (2023). "The Time-Cost Trade-off Problem and Its Extensions: A State-of-the-art Survey and Outlook." *Journal of Project Management*, **8**(3), 141–164.

Lin X, Lin Z, Wei S (2021). "Multi-objective Optimized Driving Strategy of Dual-motor EVs Using NSGA-II as a Case Study and Comparison of Various Intelligent Algorithms." *Applied Soft Computing*, **111**, 107684. doi:10.1016/j.asoc.2021.107684.

Mrad M, Al-gahtani KS, Hulchafo R, Souayah N, Bamatraf K (2019). "Risk Assessment for Discrete Stochastic Time-Cost-Quality Trade-off Problem Using Simulation-based Integer Linear Programming Approach." *IEEE Access*, **7**, 32453–32463.

Nguyen L (2021). "An Extension of Lagrange Interpolation to Approximate Derivative, Integral and Bivariate Function." doi:10.31219/osf.io/tn67p.

Pathak BK, Srivastava S (2015). "Effects of Project Uncertainties on Nonlinear Time-cost Tradeoff Profile." *Iranian Journal of Fuzzy Systems*, **12**(4), 79–100.

Pathak BK, Srivastava S, Srivastava K (2008). "Neural Network Embedded Multiobjective Genetic Algorithm to Solve Non-linear Time-Cost Tradeoff Problems of Project Scheduling." *Journal of Scientific and Industrial Research*, **67**.

Séroul R (2012). *Programming for Mathematicians*. Springer Science & Business Media.

Sharma K, Trivedi MK (2020). "AHP and NSGA II Based Time-Cost-Quality Trade-Off Optimization Model for Construction." *Second International Conference on Sustainable and Innovative Solutions for Current Challenges in Engineering & Technology (ICSISCET-2020) Madhav Institute of Technology & Science, Gwalior 18 - 19 December, 2020*. doi:10.1007/978-981-16-1220-6_5.

Sharma S, Khodadadi N, Saha AK, Gharehchopogh FS, Mirjalili S (2023). "Non-Dominated Sorting Advanced Butterfly Optimization Algorithm for Multi-objective Problems." *Journal of Bionic Engineering*, **20**(2), 819–843.

Srivastava S, Pathak B, Srivastava K (2010). "Project Scheduling: Time-Cost Tradeoff Problems." *Computational Intelligence in Optimization: Applications and Implementations*, pp. 325–357. doi:10.1007/978-3-642-12775-5-14.

Tavassoli L, Massah R, Montazeri A, Mirmozaffari M, Jiang G, Chen H (2021). "A New Multiobjective Time-Cost Trade-off for Scheduling Maintenance Problem in a Series-Parallel System." *Mathematical Problems in Engineering*, pp. 1–13. doi:10.1155/2021/5583125.

Zeng X, Cai J, Liang C, Yuan C (2023). "Prediction of Stock Price Movement Using an Improved NSGA-II-RF Algorithm with a Three-stage Feature Engineering Process." *Plos One*, **18**(6), e0287754.

Zhang W, Xiao J, Zhang S, Lin J, Feng R (2021). "A Utility-aware Multi-task Scheduling Method in Cloud Manufacturing Using Extended NSGA-II Embedded with Game Theory." *International Journal of Computer Integrated Manufacturing*, **34**(2), 175–194.

Zitzler E, Thiele L (1999). "Multiobjective Evolutionary Algorithms: A Comparative Case Study and the Strength Pareto Approach." *IEEE Transactions on Evolutionary Computation*, **3**(4), 257–271.

Affiliation:

Muskan Kaoor
Department of Mathematics
Jaypee University of Information Technology
Solan, Himachal Pradesh, India (173234)
E-mail: kapoormuskan413@gmail.com

Bhupendra Kumar Pathak*(Corresponding Author)
Department of Mathematics
Jaypee University of Information Technology
Solan, Himachal Pradesh, India (173234)
E-mail: pathak.maths@gmail.com

Rajiv Kumar
Department of Electronics and Communication Engineering
Jaypee University of Information Technology
Solan, Himachal Pradesh, India (173234)
E-mail: rjv.ece@gmail.com

Austrian Journal of Statistics http://www.ajs.or.at/
published by the Austrian Society of Statistics http://www.osg.or.at/
Volume 53 *Submitted:* 2024-02-28
2024 *Accepted:* 2024-04-01