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Induced Garima Stochastic Volatility Models

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Abstract

In the present paper, a stochastic volatility model generated by a first-order induced Garima Markov sequence has been suggested for modeling financial time series. The statistical and probabilistic properties of the suggested model are studied, and the estimation of the model parameters is done using the generalized method of moments. Simulation studies and real data analysis are done to demonstrate the applicability of the model. Also, the fit has been compared with an existing stochastic volatility model.

Keywords: generalized method of moments, induced garima, stochastic volatility.

1. Introduction

Financial time series analysis has grown quite popular in the last decade, and a lot of research has been carried out in this field. Modeling the changing conditional variance (volatility) of an asset return is the major objective of analyzing financial time series. Several models like autoregressive conditional heteroscedastic (ARCH) (Engle 1982) models and other generalizations like generalized autoregressive conditional heteroscedastic (GARCH) (Bollerslev 1986; Shephard 1996; Tsay 2005) and stochastic volatility (SV) models (Taylor 1986) have been introduced into the literature to model this time-dependent volatility. In ARCH and GARCH models, the conditional volatility is based on previous volatility and observations, whereas in the SV model, the conditional volatility is modeled by an unobservable stochastic process. The SV models capture most of the analytical properties of the financial return series and are more flexible and advanced than ARCH/GARCH models. However, they are used less frequently in empirical applications due to the difficulties in inference problems. In recent years, several estimation methods have been developed to overcome these difficulties, and the literature on SV models has grown significantly. Some of the estimation methods include the Generalized Method of Moments (GMM) (Melino and Turnbull 1990), Efficient Method of Moments (EMM) (Gallant, Hsieh, and Tauchen 1997), the Quasi Maximum Likelihood (QML) approach (Harvey, Ruiz, and Shephard 1994; Ruiz 1994), Markov-Chain Monte Carlo (MCMC) (Chib, Nardari, and Shephard 2002), the Monte Carlo Maximum Likelihood approach (Sandmann and Koopman 1998), etc.

In most of the cases, SV models are based on the assumption that the conditional distribution of the returns is normal (Jacquier, Polson, and Rossi 2002; Kim, Shephard, and Chib 1998). Later some researchers developed SV models based on non-normal conditional distributions

such as the normal inverse Gaussian distribution (Barndorff-Nielsen 1997; Andersson 2001), Student's t-distribution (Nakajima and Omori 2012), generalized error distribution (Liesenfeld and Jung 2000), generalized Student's t-distribution (Wang, Choy, and Chan 2013) etc. to account for heavy tails observed in the returns series. Another extension of the SV model is based on the assumption that the non-negative volatility sequences are generated by some non-Gaussian autoregressive process of order 1 (AR(1)) models. Abraham, Balakrishna, and Sivakumar (2006) introduced an SV model in which the volatility sequence is generated by a gamma AR(1) sequence, Balakrishna and Shiji (2014) proposed an SV model based on a first-order extreme value autoregressive model, Balakrishna and Thekkedath (2019) introduced another extension of SV model generated by first-order Birnbaum-Saunders Markov sequence, and Sri Ranganath (2018) proposed Lindley SV model.
In this paper, we propose a stationary Markov sequence of induced Garima (i-Garima) random variables (rv) to model volatilities and discuss its statistical properties and applications. The work is motivated by the Lindley SV model thoroughly discussed in Sri Ranganath (2018). The i-Garima distribution is introduced by Singh and Das (2020).

A rv X is said to follow i-Garima distribution if its probability density function (pdf) is given by,

$$f(x;\theta) = \frac{\theta}{\theta+3}(2+\theta+\theta x)e^{-\theta x}; \quad x>0, \quad \theta>0, \tag{1}$$

and the corresponding cumulative distribution function is,

$$F(x;\theta) = 1 - \left[1+\frac{\theta x}{\theta+3}\right]e^{-\theta x}; \quad x>0, \quad \theta>0.$$

The above distribution is developed using the concept of induced distribution and hence named as induced Garima distribution. The distribution is also expressed as a mixture of exponential (θ) and gamma $(2,\theta)$ with mixing proportion $q=\frac{\theta+2}{\theta+3}$.
That is,

$$f(x;\theta) = qf_1(x) + (1-q)f_2(x),$$

where $f_1(x)=\theta e^{-\theta x}$, and $f_2(x)=\theta^2 x e^{-\theta x}$.
The r^{th} order moments about the origin are given by,

$$\mu'_r = \frac{r!}{\theta^r}\frac{(\theta+r+3)}{(\theta+3)}; \quad r=1,2,3,\ldots$$

The i-Garima distribution is considered as a second-order induced Lindley distribution and has got wide applications in modeling lifetime data. However, there hasn't been much work done in the area of time series using this distribution. So in this paper, we study the application of the i-Garima distribution in time series analysis especially in modeling the volatility. The rest of the paper is organized as follows. The i-Garima SV model and its second-order properties are described in Section 2. In Section 3, we estimate the parameters of the proposed model using the generalized method of moments, and a simulation study is carried out in Section 4. In Section 5, the proposed model is illustrated using two financial data sets and compared with the Lindley SV model. Finally, the conclusions are given in Section 6.

2. The model

In this section, we introduce an SV model based on a first-order i-Garima Markov sequence and study its properties. Let R_t be the return of financial asset at time t. We define the SV model as,

$$R_t = \sqrt{h_t}\varepsilon_t, \tag{2}$$

where $\{\varepsilon_t,\ t \geq 1\}$ is a sequence of independent and identically distributed standard normal random variables. Also,

$$h_t = \phi h_{t-1} + \eta_t; \quad t = 1, 2, \ldots, \quad 0 < \phi < 1, \tag{3}$$

with the assumption that $\{h_t,\ t \geq 1\}$ follows i-Garima distribution with pdf given in (1). The sequence $\{\varepsilon_t\}$ is independent of $\{h_t\}$ and the innovation sequence $\{\eta_t\}$ for every t.

The identification of the distribution of the innovation rv η_t when h_t follows i-Garima distribution is the crucial point to be discussed first. Since $\{h_t\}$ is a stationary process with an additive structure, the Laplace transforms (LT) can be used for deriving the distribution of the innovation rv η_t.

The LT of the stationary process $\{h_t\}$ following i-Garima distribution is given by,

$$\Phi_h(s) = \frac{\theta(3\theta + \theta^2 + 2s + s\theta)}{(3+\theta)(\theta+s)^2}, \tag{4}$$

and from (3), the LT of η_t is,

$$\Phi_\eta(s) = \frac{\Phi_h(s)}{\Phi_h(\phi s)}. \tag{5}$$

Using (4) and (5), we obtain the LT of η_t as follows,

$$\Phi_\eta(s) = \frac{(3\theta + \theta^2 + 2s + s\theta)(\theta + \phi s)^2}{(3\theta + \theta^2 + 2s\phi + s\theta\phi)(\theta + s)^2}.$$

We use the partial fraction decomposition method and the properties of inverse LT to identify the distribution of η_t and obtained it as,

$$f_\eta(x) = \phi\delta(x) + (1-\phi)r(x). \tag{6}$$

Hence, η_t is derived as a mixture of two distributions, where $\delta(x)$ is the Dirac Delta function defined by,

$$\delta(x) = \begin{cases} +\infty & if \quad x = 0, \\ 0 & if \quad x \neq 0, \end{cases}$$

and $r(x)$ has the pdf,

$$\begin{aligned} r(x) = &\left(\frac{3\theta^2\phi^2 - \theta^2\phi^3 - 3\theta^2\phi + \theta^2 - 3\theta\phi^3 + 11\theta\phi^2 - 13\theta\phi + 5\theta - 2\phi^3 + 8\phi^2 - 12\phi + 6}{(1-\phi)(\theta(1-\phi) + 3 - 2\phi)^2}\right)\theta\exp(-\theta x) \\ &+ \frac{1-\phi}{(\theta(1-\phi) + 3 - 2\phi)}\theta^2 x\exp(-\theta x) \\ &- \frac{\phi}{(\theta(1-\phi) + 3 - 2\phi)^2}\left(\frac{\theta^2 + 3\theta}{2\phi + \theta\phi}\right)\exp\left(-\frac{\theta^2 + 3\theta}{2\phi + \theta\phi}\right)x, \end{aligned}$$

with $\theta > 0, \quad x \geq 0$. It is clearly found that $r(x)$ is a mixture of exponential (θ), gamma $(2, \theta)$ and exponential $(\frac{\theta^2 + 3\theta}{2\phi + \theta\phi})$.

The mean and variance of η_t are respectively,

$$E(\eta_t) = \frac{\theta + 4}{\theta(\theta+3)}(1-\phi), \tag{7}$$

and

$$Var(\eta_t) = \frac{(1-\phi^2)(\theta^2 + 8\theta + 14)}{\theta^2(\theta+3)^2}. \tag{8}$$

Next, we study the statistical properties of the proposed SV model. On considering the return series, it is obvious from the model in (2) that the odd moments of $\{R_t\}$ are zero and its even moments are given by,

$$\begin{aligned} E\left(R_t^{2r}\right) &= E\left(\varepsilon_t^{2r}\right) E\left(h_t^r\right) \\ &= \left(\frac{r!(\theta+r+3)}{\theta^r(\theta+3)}\right)\prod_{j=1}^{r}(2j-1); \quad r = 1,2,\ldots \end{aligned}$$

The second-order moments are,

$$E\left(R_t^2\right) = Var\left(R_t\right) = \frac{\theta+4}{\theta(\theta+3)},$$

and

$$E\left(R_t^2 R_{t-1}^2\right) = \frac{\phi(\theta^2+8\theta+14)+(\theta+4)^2}{\theta^2(\theta+3)^2}.$$

We have the variance and covariance of the squared return series given by,

$$Var\left(R_t^2\right) = \frac{5\theta^2+40\theta+74}{\theta^2(\theta+3)^2},$$

and

$$\gamma_k\left(R_t^2\right) = \phi^{|k|}\frac{\theta^2+8\theta+14}{\theta^2(\theta+3)^2}.$$

The kurtosis of the returns R_t is,

$$K_R = 6\frac{(\theta^2+8\theta+15)}{\theta^2+8\theta+16} > 3, \tag{9}$$

which implies that the distribution is leptokurtic. In most cases, kurtosis associated with returns is a reflection of the time-varying property of volatility. By choosing different values of θ in (9), we get a distribution with more flexible kurtosis (see Figure 1).

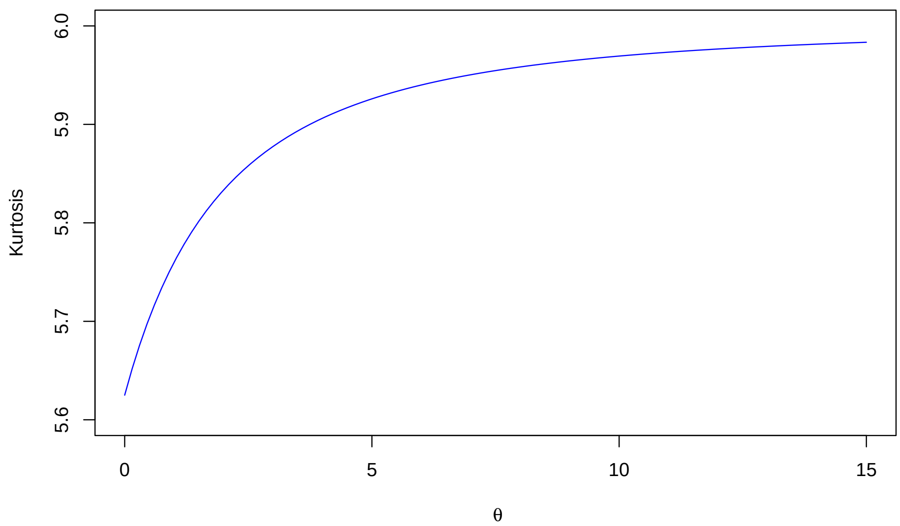


Figure 1: The plot of kurtosis of return R_t

The structure of the model in (2) indicates that autocorrelation function (ACF) of $\{R_t\}$ is zero and that of $\{R_t^2\}$ is significant. The ACF of $\{R_t^2\}$ is given by,

$$\begin{aligned}\rho_{R_t^2}(k) &= Corr\left(R_t^2, R_{t-k}^2\right)\\ &= \phi^{|k|}\frac{\theta^2+8\theta+14}{5\theta^2+40\theta+74}.\end{aligned}$$

From Figure 2, it is found that for different values of parameters, the ACF is exponentially decreasing function of lags.

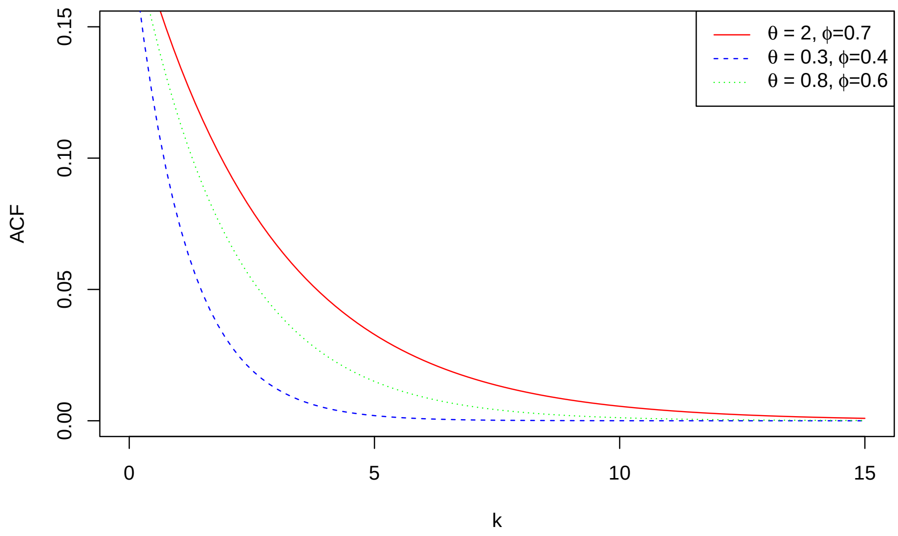


Figure 2: The ACF of squared returns for different combinations of the parameters

In the next section, we discuss the estimation of the unknown parameters of the proposed model.

3. Estimation of the parameters

The likelihood-based inference for the SV model is difficult due to the presence of unobservable Markov-dependent latent variables in the likelihood function. To integrate these latent variables, many techniques based on MCMC methods have been proposed in the literature. In this paper, we use the GMM proposed by Hansen (1982) for estimation. We have also used the steps given in Sri Ranganath (2018) for estimation purposes and in the coming section of the simulation.

Let $(R_1, R_2, \ldots, R_N)$ be the observed return series of length N from i-Garima SV model (2) and let the parameter vector to be estimated be $\Theta = (\phi, \theta)'$. In GMM, we equate the moments $E\left(R_t^2\right)$ and $E\left(R_t^2 R_{t-1}^2\right)$ to sample moments $\bar{R}_2 = (1/N)\sum_{t=1}^N R_t^2$ and $\bar{R}_{22} = (1/N)\sum_{t=1}^N R_t^2 R_{t-1}^2$ respectively and the resulting moment equations for ϕ and θ are obtained as,

$$\bar{R}_2 = \frac{\hat{\theta}+4}{\hat{\theta}(\hat{\theta}+3)}, \quad \hat{\phi} = \frac{\bar{R}_{22}\left(\hat{\theta}^2(\hat{\theta}+3)^2\right)-(\hat{\theta}+4)^2}{\hat{\theta}^2+8\hat{\theta}+14}. \tag{10}$$

The asymptotic properties of the generalized moment estimators, under certain assumptions, are studied by Hansen (1982). Now, we state the following result for the proposed i-Garima SV model using these regularity conditions.

Theorem 1 *Suppose the sequence* $\{R_t : -\infty < t < \infty\}$ *satisfies the assumptions stated by Hansen (1982). Then* $\{\sqrt{N}(\hat{\Theta} - \Theta), N \geq 1\}$ *converges in distribution to a normal random vector with mean 0 and dispersion matrix* $\left[DS^{-1}D'\right]^{-1}$*, where* $D = E\left[\partial h\left(R_t, \Theta\right)/\partial\Theta\right]$*,* $S = \sum_{k=-\infty}^{\infty} \Gamma_{(k)}$*,* $\Gamma_{(k)} = E\left(\omega_t \omega_{t-k}'\right)$ *and* $\omega_t = h\left(R_t, \Theta\right), -\infty < t < \infty$*.*

The asymptotic dispersion matrix of $\hat{\Theta}$ appeared above is derived as follows.
Since h_t is ergodic, stationary, and has finite moments, the sequence $\{R_t\}$ given in (2) also hold these properties. As a result, our i-Garima SV model satisfies the regularity criterion specified above. We derive the elements of the proposed model's dispersion matrix in order to determine the estimators' asymptotic standard errors.

Let $\Gamma_{(k)} = \begin{pmatrix} \gamma_{11}^{(k)} & \gamma_{12}^{(k)} \\ \gamma_{21}^{(k)} & \gamma_{22}^{(k)} \end{pmatrix}$, $k = 0, \pm 1, \pm 2, \ldots$ with $\Gamma_{(k)} = \Gamma_{(-k)}$, $k = 1, 2, \ldots$. Then the matrix S is obtained by $S = \Gamma_{(0)} + 2\sum_{k=1}^{\infty} \Gamma_{(k)}$.

For $k = 0$, we found the elements of $\Gamma_{(0)} = E\left(\omega_t \omega_t'\right)$ as,

$$\gamma_{11}^{(0)} = \frac{5\theta^2 + 40\theta + 74}{\theta^2(\theta+3)^2},$$
$$\gamma_{12}^{(0)} = \gamma_{21}^{(0)} = \frac{A_1}{\theta^3(\theta+3)^2},$$
$$\gamma_{22}^{(0)} = \frac{216\phi^2(\theta+7)(\theta+3)^2 + 108\phi(1-\phi)(\theta+4)(\theta+6)(\theta+3) + A_2}{\theta^4(\theta+3)^3} - A_3^2,$$

where

$$A_1 = 296 + 234\theta + 60\theta^2 + 5\theta^3 + 280\phi + 23\theta\phi + 60\theta^2\phi + 5\theta^3\phi + 276\phi^2 + 252\theta\phi^2 + 72\theta^2\phi^2 + 6\theta^3\phi^2,$$

$$A_2 = 18(\theta+5)\left((1-\phi^2)(\theta^2 + 8\theta + 14) + (\theta+4)^2(1-\phi)^2\right),$$

and $$A_3 = \frac{\phi(\theta^2 + 8\theta + 14) + (\theta+4)^2}{\theta^2(\theta+3)^2}.$$

For $k = 1, 2, \ldots$ the elements of $\Gamma_{(k)}$ are,

$$\gamma_{11}^{(k)} = \phi^{|k|}\frac{\theta^2 + 8\theta + 14}{\theta^2(\theta+3)^2},$$
$$\gamma_{12}^{(k)} = \phi^{k+2}a_3 + 2\phi^{k+1}a_2b_1 + \phi^k a_1 b_2 + a_2 b_1 \sum_{i=0}^{k-1}\phi^{i+1} + a_1 b_1^2 \sum_{i=0}^{k-1}\phi^i - \phi a_1 a_2 - a_1^2 b_1,$$
$$\gamma_{21}^{(k)} = \phi^{2k-1}a_3 + \phi^k a_2 b_1 \sum_{i=0}^{k-2}\phi^i + \phi^{k-1}a_2 b_1 \sum_{i=0}^{k-1}\phi^i + a_1 b_2 \phi^{2k-3} + a_1 b_1^2 \sum_{i=0}^{k-1}\phi^i \sum_{i=0}^{k-2}\phi^i$$
$$- \phi a_1 a_2 - a_1^2 b_1,$$
$$\gamma_{22}^{(k)} = \phi^{2k+2}a_4 + a_3 b_1\left(3\phi^{2k+1} + \phi^{k+1} + 2\sum_{i=1}^{k-1}\phi^{i+k+1}\right) + \phi^{2k}\left(3a_2b_2 + a_1b_3\right)$$
$$+ \sum_{i=0}^{k-1}\phi^i \sum_{i=0}^{k-2}\phi^i(a_1 b_1^3 + \phi a_2 b_1^2) + a_2 b_1^2\left(\phi^k + 2\sum_{i=1}^{k-1}\phi^{i+k}\right) - \phi a_1 a_2 - a_1^2 b_1,$$

where

$$a_1 = \frac{\theta+4}{\theta(\theta+3)}, \quad a_2 = \frac{2\left(\theta+5\right)}{\theta^2(\theta+3)}, \quad a_3 = \frac{6\left(\theta+6\right)}{\theta^3(\theta+3)},$$

$$b_1 = \frac{\theta+4}{\theta(\theta+3)}(1-\phi), \quad b_2 = \frac{(1-\phi^2)(\theta^2+8\theta+14) + (\theta+4)^2(1-\phi)^2}{\theta^2(\theta+3)^2},$$

and $b_3 = E(\eta_t^3)$.

The matrix D is obtained using $D = E\left[\partial h\left(R_t, \Theta\right)/\partial\Theta\right]$ and its corresponding elements are,

$$D_{11} = \frac{\theta^2 + 8\theta + 12}{\theta^2(\theta+3)^2},$$
$$D_{12} = \frac{-96 - 88\theta - 24\theta^2 - 2\theta^3 - 84\phi - 80\theta\phi - 24\theta^2\phi - 2\theta^3\phi}{\theta^3(\theta+3)^3},$$
$$D_{21} = 0,$$
$$D_{22} = -\frac{\theta^2 + 8\theta + 4}{\theta^2(\theta+3)^2}.$$

The asymptotic dispersion matrix is, therefore, $\frac{1}{N}(\Sigma)$, where $\Sigma = \left[DS^{-1}D'\right]^{-1}$ and the diagonal elements of Σ are used to compute the asymptotic standard errors of the estimators.

4. Simulation study

We conduct a simulation analysis with samples of sizes 100, 500, 2000, and 3000 to understand the performance of the estimators. For assessing the accuracy of the performance of the estimators, each sample is generated 1000 times for certain fixed values of the parameters. The innovation rv defined in (6) is used to generate a sample from the i-Garima Markov sequence. In the next step, we simulate a sample of returns R_t using this. The moment equations given in (10) are solved to obtain the estimates. On the basis of these simulated observations, the mean estimates and their associated root mean square errors (RMSE) are presented in Table 1. It is found that the bias and the RMSE values are decreasing with an increase in sample size ensuring the stability of the proposed estimation method.

Table 1: The estimates and the corresponding RMSE of the parameters for sample sizes 100, 500, 2000, and 3000

	True Values		Mean estimate values			
Sample size(n)	θ	ϕ	$\hat{\theta}$	RMSE	$\hat{\phi}$	RMSE
100	0.2	0.30	0.2072	0.0427	0.2968	0.0043
	0.1	0.60	0.1048	0.0230	0.5935	0.0085
	0.5	0.35	0.5169	0.1096	0.3465	0.0048
	0.7	0.50	0.7332	0.1579	0.4947	0.0072
	0.8	0.45	0.8439	0.1789	0.4455	0.0060
500	0.2	0.30	0.2010	0.0183	0.2994	0.0007
	0.1	0.60	0.1006	0.0098	0.5987	0.0016
	0.5	0.35	0.5071	0.0499	0.3492	0.0009
	0.7	0.50	0.7065	0.0678	0.4990	0.0013
	0.8	0.45	0.8076	0.0751	0.4490	0.0012
2000	0.2	0.30	0.2002	0.0097	0.2998	0.0002
	0.1	0.60	0.0999	0.0048	0.5996	0.0004
	0.5	0.35	0.5010	0.0246	0.3498	0.0002
	0.7	0.50	0.6999	0.0323	0.4997	0.0003
	0.8	0.45	0.8030	0.0378	0.4497	0.0003
3000	0.2	0.30	0.1998	0.0078	0.2999	0.0001
	0.1	0.60	0.1003	0.0039	0.5997	0.0002
	0.5	0.35	0.5002	0.0183	0.3498	0.0001
	0.7	0.50	0.6996	0.0266	0.4998	0.0002
	0.8	0.45	0.8011	0.0298	0.4498	0.0002

5. Data analysis

We illustrate the application of the proposed i-Garima SV model using two real data sets:

1. Daily returns for the rate of exchange on Euro to US Dollar (EURUSD) for the period starting from January 1, 2010 to January 31, 2020 (https://www.marketwatch.com).

2. The opening index data of US Dollar Index future (DXY) for the period January 3, 2005 to December 31, 2020 (https://www.marketwatch.com).

We denote the daily price index by p_t. The returns are then obtained by the transformation,

$$R_t = 100\left(\log\left(\frac{p_t}{p_{t-1}}\right) - \frac{1}{n}\sum_{t=1}^{n}\log\left(\frac{p_t}{p_{t-1}}\right)\right); \quad t = 1, \dots, n.$$

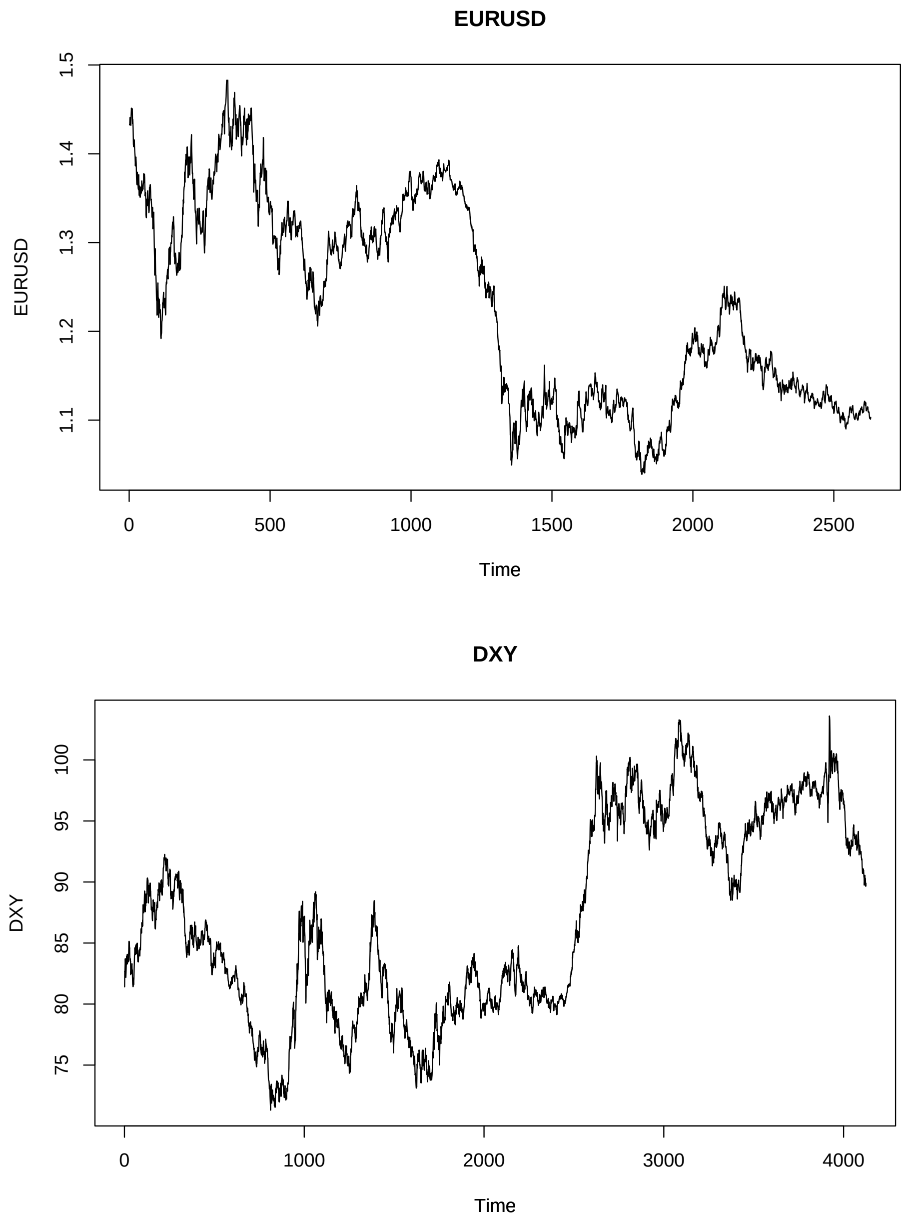


Figure 3: Time series plots of the original data

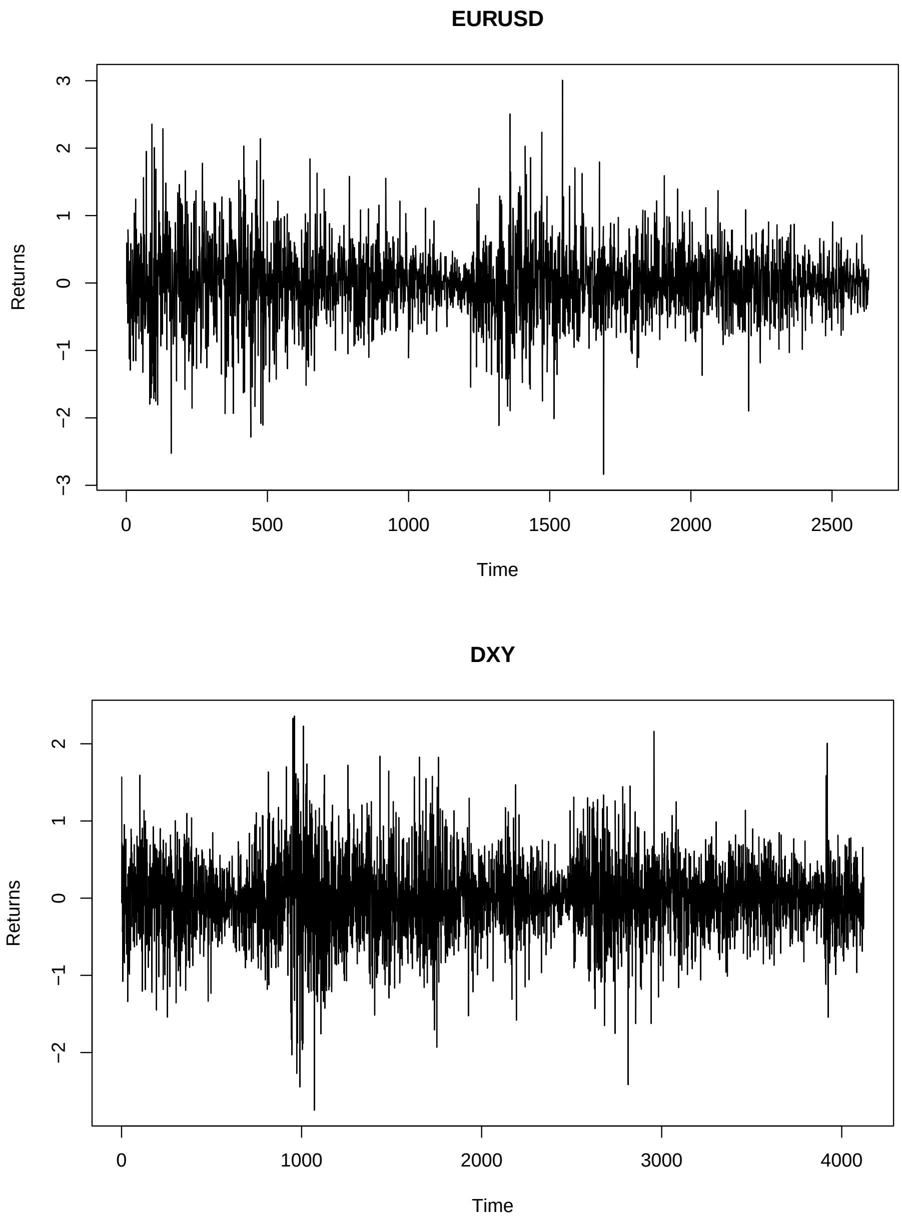


Figure 4: Time series plots of the returns

The time series plots of the original series and returns are exhibited in Figures 3 and 4, respectively. The summary statistics for the return series are computed in Table 2. By the Ljung–Box portmanteau test, it is clearly confirmed that the presence of serial correlations in returns is rejected with a p-value of 0.3351 (EURUSD) and 0.1708 (DXY) and in squared returns, the absence of autocorrelations is rejected by the same test with a p-value less than 0.05 for both the data sets.

The kurtosis is greater than 3 for both series, indicating that the returns have a leptokurtic distribution. Also, the plot of ACF of returns (Figure 5) shows that the serial correlations in the return series are insignificant whereas that of the squared returns series (Figure 6) are significant and decay very slowly.

Table 2: Summary statistics of the return series

Statistics	EURUSD	DXY
Sample size	2630	4123
Std. Dev	0.5668	0.4893
Minimum	−2.8392	−2.7473
Maximum	3.0071	2.3599
Kurtosis	5.0559	5.0598
Ljung-Box p-value for returns.	0.3351	0.1708
Ljung-Box p-value for squared returns.	2.2×10^{-16}	2.2×10^{-16}

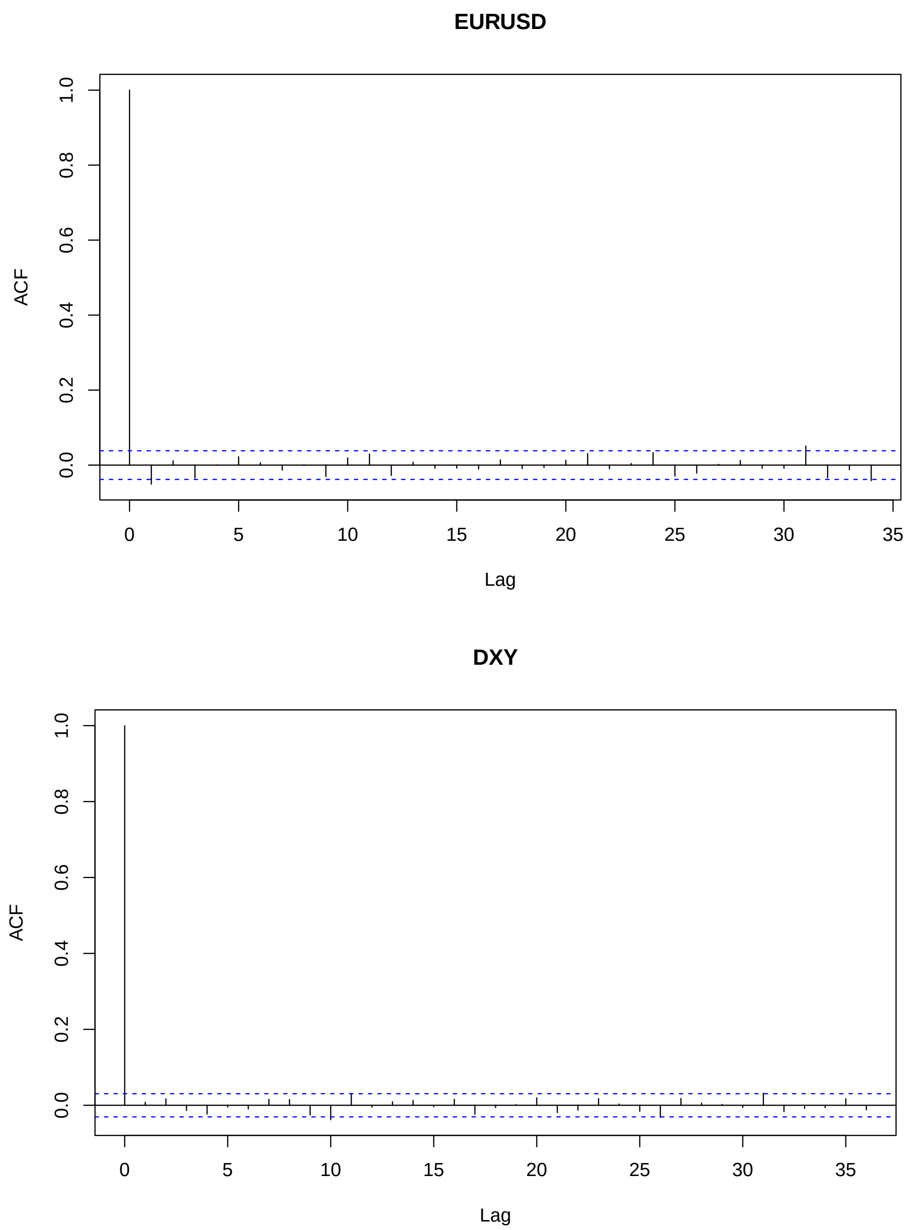


Figure 5: ACF of the returns

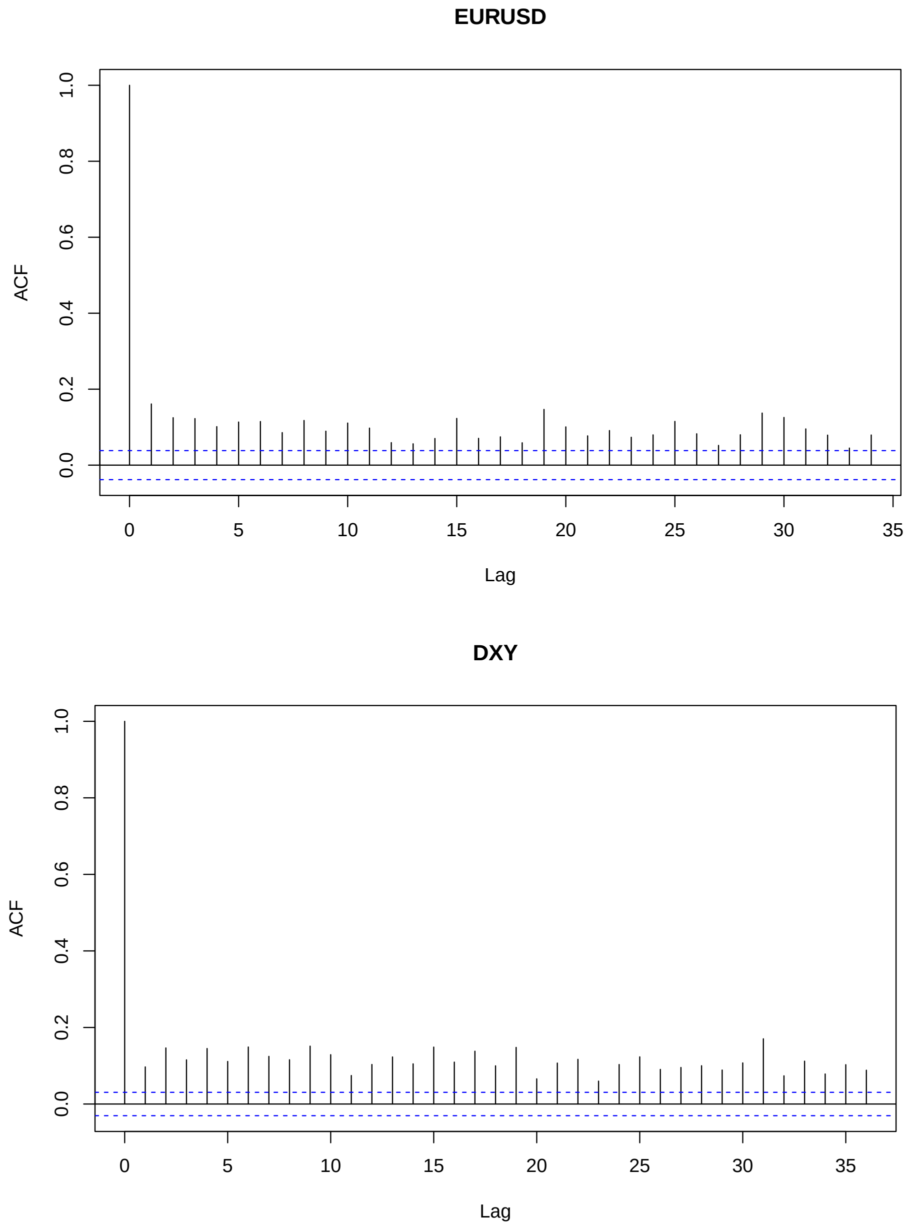


Figure 6: ACF of the squared returns

Here, we also compare the i-Garima SV model with the Lindley SV model (Sri Ranganath 2018) for the above data sets. Now, as mentioned in Section 3, we estimate the model parameters of both series using the GMM, and the parameter estimates are presented in Table 3 for both i-Garima and Lindley SV models. The values of $\hat{\phi}$ in Table 3 indicate that in the above data sets there is a significant persistence of volatility. To compare the models, the values of the Kolmogorov-Smirnov (K-S) statistic for the above data sets are also computed. It is observed that compared to the Lindley SV model, the i-Garima SV model provides slightly higher p-values corresponding to the K-S test (see Table 3). This indicates that the proposed i-Garima SV model is an equally competitive and better alternative model for the Lindley SV model.

The next step after estimating the parameters is model diagnostic checking. That is, we need to examine whether the assumptions of the model in (2) are satisfied with the data we have analyzed. Our model (2) is expressed in terms of unobservable volatilities h_t which makes the diagnosis problem difficult. In such instances, one way proposed is to use Kalman filtering by

Table 3: Estimates of the model parameters and the K-S statistic values for the data sets

Data Set	Model	Estimates $\hat{\theta}$	$\hat{\phi}$	K-S	p-value
EURUSD	i-Garima SV Model	3.5861	0.6779	0.0317	0.0787
	Lindley SV Model	3.7666	0.6963	0.0338	0.0503
DXY	i-Garima SV Model	4.7181	0.4023	0.0237	0.1945
	Lindley SV Model	4.8865	0.4090	0.0291	0.0608

rewriting the model (2) in state-space form (Jacquier *et al.* 2002; Tsay 2005). This method can be used to estimate the unobservable volatility h_t by approximating the distribution of η_t specified in (3) by a normal distribution. Then, we can compute the residuals using these estimated volatilities.
The state space representation of the i-Garima SV model is,

$$\log R_t^2 = -1.27 + \log h_t + \tau_t, \quad E(\tau_t) = 0, \quad Var((\tau_t) = \frac{\pi^2}{2}, \tag{11}$$

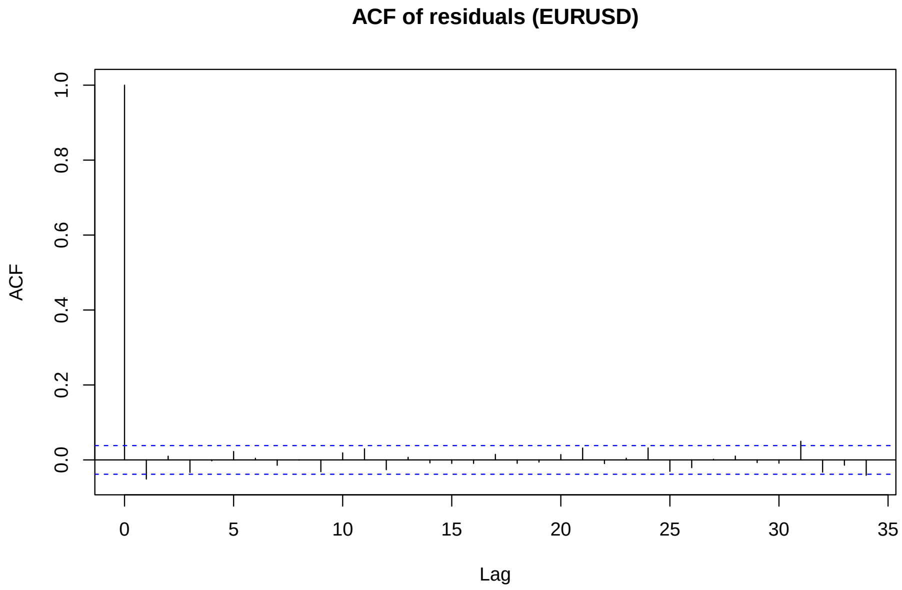
and $h_t = \phi h_{t-1} + \eta_t$, where η_t is assumed to be normally distributed with mean and variance given in (7) and (8), respectively. By presuming normal distribution to approximate τ_t, (11) becomes a standard dynamic linear model. Then Kalman filtering can be used under this setup. Let $\bar{h}_{t|t-1}$, $\Lambda_{t|t-1}$, $\bar{h}_{t|t}$, $\Lambda_{t|t}$ represent the prediction of h_t and its variance at time $t-1$ and t respectively. The predictions are then computed and updated iteratively using the equations,

$$\bar{h}_{t|t-1} = \phi \bar{h}_{t-1|t-1} + \frac{(\theta+4)(1-\phi)}{\theta(\theta+3)},$$
$$\Lambda_{t|t-1} = \phi^2 \Lambda_{t-1|t-1} + \frac{(1-\phi^2)(\theta^2+8\theta+14)}{(\theta^2(\theta+3)^2)},$$

and

$$\bar{h}_{t|t} = \bar{h}_{t|t-1} + \frac{\Lambda_{t|t-1}}{g_t}\left[\log R_t^2 + 1.27 - \log \bar{h}_{t|t-1}\right],$$
$$\Lambda_{t|t} = \Lambda_{t|t-1}\left(1 - \frac{\Lambda_{t|t-1}}{g_t}\right),$$

where $g_t = \Lambda_{t|t-1} + \frac{\pi^2}{2}$.
The residuals are obtained from the equation $\hat{\varepsilon}_t = \frac{R_t}{\sqrt{\bar{h}_t}}$ and the residual analysis is carried out using this sequence. The initial values are taken as $\Lambda_0 = \frac{(\theta^2+8\theta+14)}{(\theta^2(\theta+3)^2)}$ and $h_0 = \frac{(\theta+4)}{\theta(\theta+3)}$ and the parameters θ and ϕ in the above system are replaced by their respective estimates $\hat{\theta}$ and $\hat{\phi}$. The plots of ACF of the residuals of both data sets are given in Figure 7 indicating the absence of significant serial correlations. This is confirmed using the Ljung–Box test, and the value of the test statistic for residual series is 12.271 with a p-value of 0.9064 (EURUSD) and 25.05 with a p-value of 0.1995 (DXY).



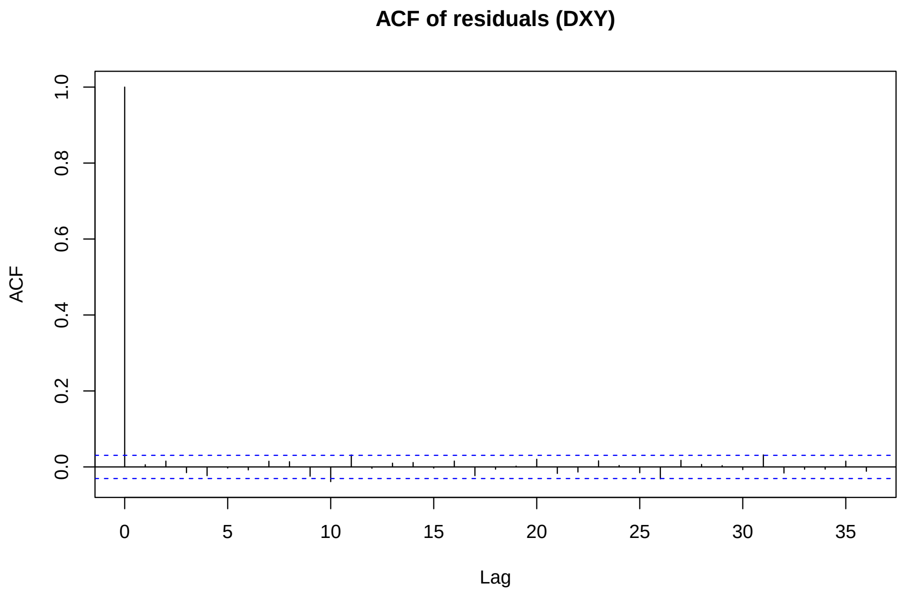


Figure 7: ACF of the residuals

Finally, we check whether the residual series follows standard normal distribution by superimposing the histogram of residuals by the density of standard normal variate. From Figure 8, it is found that normal distribution is a good fit for residuals in both cases. Thus, we have analyzed the model's adequacy and observed that it performs satisfactorily.

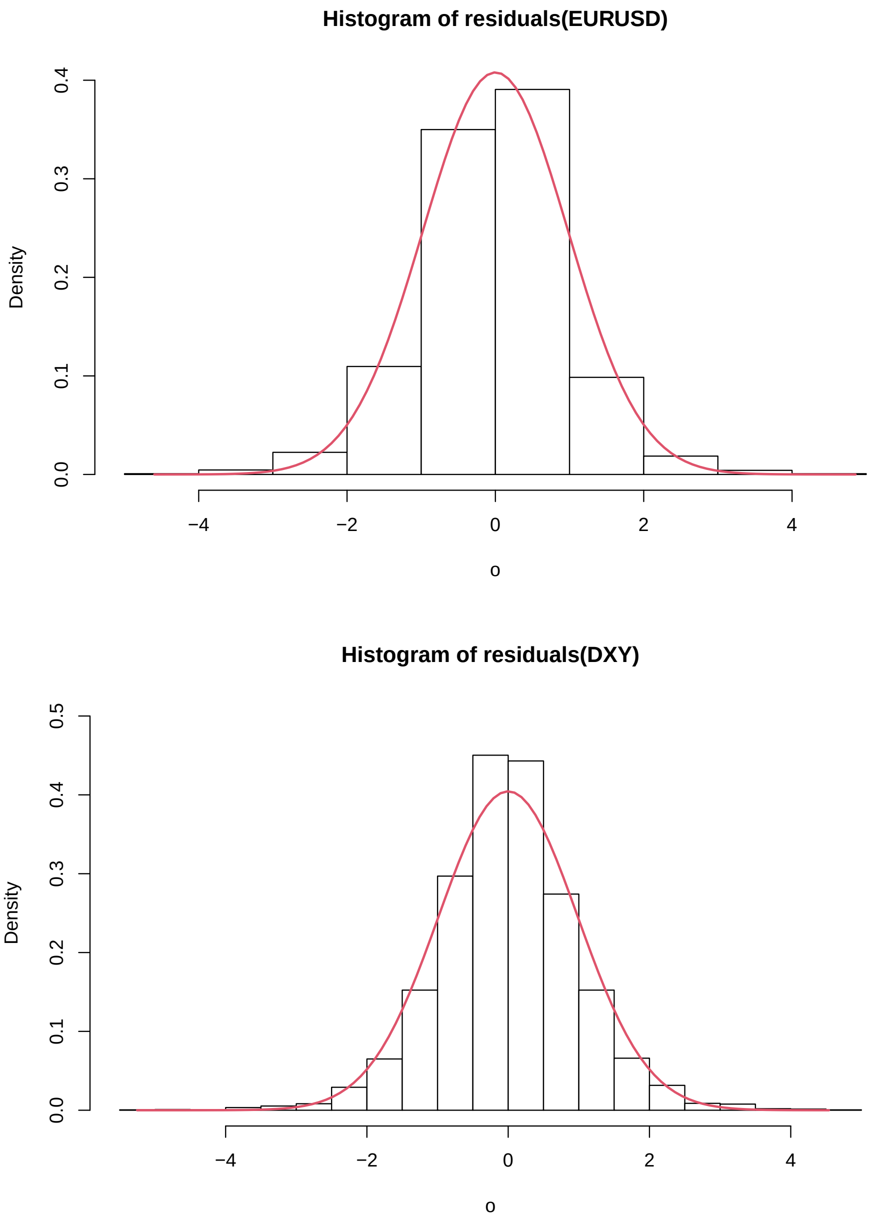


Figure 8: Histogram of residuals with superimposed standard normal density

6. Conclusion

In the present paper, a stationary sequence of induced Garima random variables is proposed to model stochastic volatility for analyzing financial time series. This could be a viable alternative to the Lindley SV model. The method of moments is effective for estimating parameters and an approximate Kalman filtering method is used to diagnose the model. The simulation and the data analyses show that the suggested model is capable of capturing the key characteristics of financial time series. However, the inference technique can be improved to gain a better understanding of the applications of the proposed model.

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