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Bayes Prediction Bound Lengths under Different Censoring Criterion: A Two-Sample Approach

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Abstract

The censoring arises when exact lifetimes are known partially only, and it is useful in life testing experiments for time and cost restrictions. In literature, there are several types of censoring plans available. In which three different censoring plans have addressed in the present comparative study. The Burr Type-XII distribution considered here as the underlying model and the comparison made on Two-Sample Bayes prediction bound lengths. The analysis of the present discussion has carried out by a real life example and simulated data both.

Keywords: Burr Type-XII distribution, two-sample plan, Type-II censoring, right censoring, progressive Type-II right censoring, Bayes prediction bound length.

1. Introduction

The cumulative density and probability density function of Burr Type-XII distribution are given as

$$F(x;\theta,\sigma) = 1 - (1 + x^{\sigma})^{-\theta} \; ; \; \theta > 0, \sigma > 0, x \geq 0 \tag{1}$$

and

$$f(x;\theta,\sigma) = \sigma\theta x^{\sigma-1}(1 + x^{\sigma})^{-\theta-1} \; ; \; \theta > 0, \sigma > 0, x \geq 0. \tag{2}$$

The two-parameter Burr Type-XII distribution has unimodal or decreasing failure rate function

$$\rho(x) = \sigma\theta x^{\sigma-1}(1 + x^{\sigma})^{-1} \; ; \; \theta > 0, \sigma > 0, x \geq 0 \tag{3}$$

The shape of the failure rate function $\rho(x)$ does not affected by the parameter θ. The parameter θ and σ both are known as shape parameter. Also, $\rho(x)$ has a unimodal curve when $\sigma > 1$ and it has decreased failure rate function when $\sigma \leq 1$. The Burr Type-II distribution is applied in several areas including study of quality control and reliability, duration study and failure time modeling. The analysis of business failure data, the efficacy of analgesics in clinical trials, and the times to failure of electronic components are the other areas of application of the said distribution. Zimmer, Keats, and Wang (1998) discussed at several statistical properties of the underlying distribution based on reliability analysis.

El-Sagheer (2016) discussed in his recent paper, about the point and interval predictions based on general progressive Type-II censored data by using generalized Pareto distribution under Bayesian setup for two-sample prediction approach. Rao, Aslam, and Kundu (2015) discuss about the multi-component stress strength reliability based on ML estimation criteria by assuming Burr Type-XII distribution in his recent paper. Using Koziol-Green model of random censorship Danish and Aslam (2014) deals the Bayes estimation for unknown parameters of the underlying distribution by assuming both the informative and non-informative priors. Jang, Jung, Park, and Kim (2014) discussed some estimation based on Bayesian setup for Burr Type-XII distribution under progressive censoring.

Soliman, Abd-Ellah, Abou-Elheggag, and Modhesh (2012) obtained some Bayes estimation from Burr Type-XII distribution by using progressive first-failure censored data. Lee, Wu, and Hong (2009) obtained Bayes and empirical Bayes estimators of reliability parameters under progressively Type-II Burr censored samples. Many works have done on underlying distribution, a little few of them discussed above, and a few more are Rodriguez (1977), Nigm (1988), Al-Huesaini and Jaheen (1995), Ali-Mousa and Jaheen (1998), Wu and Yu (2005), El-Sagheer and Ahsanullah (2015), Soliman, Abd-Ellah, Abou-Elheggag, and El-Sagheer (2015) and El-Sagheer (2016).

It is not always possible that the experimentally observed the lifetimes of all inspected units in life testing experiments, due to time limitation and/or cost or material resources for data collection. In addition, when some sample values at either or both extremes adulterated, the trimmed samples are useful. There are several types of censoring plans available in literature, in which only three common censoring plans have addressed in the present study.

The article presents a comparative study under Two-Sample Bayes prediction bounds length by using different censoring plans, wiz, Item-Failure, right Item-Failure, and Progressive Type-II censoring. The Bayes prediction bounds lengths have obtained from the underlying model. The properties of the procedures are illustrated by simulated data as well as a real data set.

2. Bayes prediction bound lengths (Two-sample technique)

When sufficient information regarding the past and the present behavior of an observation is available, we predict the nature of the future behavior of an observation in the present section. A Bayesian statistical analysis has applied here for predicting future statistic from the model given in Eq. (2), based on all three considered censoring plans.

Let $x_{(1)}, x_{(2)}, ..., x_{(r)}$ be the first r observed ordered failure items from a sample of size n under considered censoring scheme for the model Eq. (2). If $y_{(1)}, y_{(2)}, ..., y_{(k)}$ is the second (unobserved) items censored data of size k drawn independently from the same model of size N, then the first sample is known as informative sample, while the second sample is referred to as future sample. Our aim is to predict the j^{th} order statistic in the future sample based on an informative sample. This prediction technique is known as the, Two-sample Bayes prediction technique. Recently, Prakash and Singh (2013) discussed about the Bayes prediction limits under two-sample plan for the Pareto model.

2.1. Item-failure censoring

Let us suppose a total of n items from considering model are put under the life test and the test terminates when first $r^{th} (r \leq n)$ item fails. This censoring scheme is known as Item-Failure censoring scheme. In such test situations, the observations usually occurred in ordered of weakest items failed first.

Let us assume that $x_{(1)}, x_{(2)}, ..., x_{(n)}$ be n ordered items from Eq. (2). If $\underline{x} \cong \left(x_{(1)}, x_{(2)}, ..., x_{(r)}\right)$ be first r observed failure items, then the joint probability density function for these order statistics is defined as

$$f_I\left(\underline{x}|\theta,\sigma\right) = \left(\prod_{i=1}^{r} f\left(x_{(i)};\theta,\sigma\right)\right)\left(1 - F\left(x_{(r)};\theta,\sigma\right)\right)^{n-r}$$

$$= \left(\prod_{i=1}^{r} \sigma\theta x_{(i)}^{\sigma-1}\left(1+x_{(i)}^{\sigma}\right)^{-\theta-1}\right)\left(1+x_{(r)}^{\sigma}\right)^{-\theta(n-r)}$$

$$\Rightarrow f_I\left(\underline{x}|\theta,\sigma\right) \propto \theta^r exp\left(-\theta T_I\left(\underline{x};\theta,\sigma\right)\right)\ ; \tag{4}$$

where $T_I\left(\underline{x};\theta,\sigma\right) = \sum_{i=1}^{r} log\left(1+x_{(i)}^{\sigma}\right) + (n-r)log\left(1+x_{(r)}^{\sigma}\right)$.

There is no honest way to define, which prior probability estimate is better. Based on personal beliefs, one may choice a flexible family of priors, and chosen one from that family, which matches best. In the present study, Gamma distribution $G(1,\alpha)$ taken as the conjugate family of prior for unknown parameter θ, with the probability density function

$$\pi_\theta = \alpha e^{-\alpha\theta}\ ;\ \alpha > 0, \theta > 0. \tag{5}$$

Based on Bayes theorem, the posterior density about the parameter θ under considered censoring plan is defined as

$$\pi_{I\theta}^{*} = \frac{f_I\left(\underline{x}|\theta,\sigma\right)\cdot\pi_\theta}{\int_\theta f_I\left(\underline{x}|\theta,\sigma\right)\cdot\pi_\theta\, d\theta}. \tag{6}$$

Using Eq. (4) and Eq. (5) in Eq. (6), the posterior density is now obtained as

$$\pi_{I\theta}^{*} \propto \frac{\theta^r exp\left(-\theta T_I\left(\underline{x};\theta,\sigma\right)\right)\cdot e^{-\alpha\theta}}{\int_\theta \theta^r exp\left(-\theta T_I\left(\underline{x};\theta,\sigma\right)\right)\cdot e^{-\alpha\theta}\, d\theta}$$

$$\Rightarrow \pi_{I\theta}^{*} = \frac{\left(T_I^{*}\left(\underline{x};\theta,\sigma\right)\right)^{r+1}}{\Gamma(r+1)}\theta^r exp\left(-\theta T_I^{*}\left(\underline{x};\theta,\sigma\right)\right)\ ;\ T_I^{*}\left(\underline{x};\theta,\sigma\right) = T_I\left(\underline{x};\theta,\sigma\right) + \alpha. \tag{7}$$

The Bayes predictive density of future observation Y is denoted by $h_I\left(Y|\underline{x}\right)$ and obtained by simplifying the following relation

$$h_I\left(Y|\underline{x}\right) = \int_\theta f_I\left(y;\theta,\sigma\right)\cdot\pi_{I\theta}^{*}\, d\theta$$

$$\Rightarrow h_I\left(Y|\underline{x}\right) = (r+1)\sigma y^{\sigma-1}\left(1+y^{\sigma}\right)^{-1}\frac{\left(T_I^{*}\left(\underline{x};\theta,\sigma\right)\right)^{r+1}}{\left(T_I^{*}\left(\underline{x};\theta,\sigma\right)+log\left(1+y^{\sigma}\right)\right)^{r+2}}. \tag{8}$$

Based on predictive density Eq. (8) of the future observation Y, the cumulative predictive density function is denoted as $G_I\left(Y|\underline{x}\right)$ and obtained as

$$G_I\left(Y|\underline{x}\right) = Pr\left(Y \le y\right)$$

$$= \left(T_I^{*}\left(\underline{x};\theta,\sigma\right)\right)^{r+1}(r+1)\sigma\int_0^y \frac{y^{\sigma-1}\left(1+y^{\sigma}\right)^{-1}}{\left(T_I^{*}\left(\underline{x};\theta,\sigma\right)+log\left(1+y^{\sigma}\right)\right)^{r+2}}dy$$

$$G_I\left(Y|\underline{x}\right) = 1 - \left(\frac{T_I^*\left(\underline{x};\theta,\sigma\right)}{T_I^*\left(\underline{x};\theta,\sigma\right) + log\left(1+y^{\sigma}\right)}\right)^{r+1}. \tag{9}$$

Now, if Y_j denote the j^{th} order statistic in future sample of size $k; 1 \leq j \leq k$, then from k future observations, the probability density function of the j^{th} ordered future observation is given as

$$\Phi_I\left(y_j\right) = j\left({}^kC_j\right)\left(G_I\left(Y_j|\underline{x}\right)\right)^{j-1}\left(1 - G_I\left(Y_j|\underline{x}\right)\right)^{k-j} h_I\left(Y_j|\underline{x}\right)$$

$$\Rightarrow \Phi_I\left(Y_j\right) = j\left({}^kC_j\right)\left(1 - \left(\frac{T_I^*\left(\underline{x};\theta,\sigma\right)}{T_I^*\left(\underline{x};\theta,\sigma\right) + log\left(1+y_j^{\sigma}\right)}\right)^{r+1}\right)^{j-1}$$

$$\cdot\left(\left(\frac{T_I^*\left(\underline{x};\theta,\sigma\right)}{T_I^*\left(\underline{x};\theta,\sigma\right) + log\left(1+y_j^{\sigma}\right)}\right)^{r+1}\right)^{k-j}$$

$$\cdot(r+1)\sigma y_j^{\sigma-1}\left(1+y_j^{\sigma}\right)^{-1}\frac{\left(T_I^*\left(\underline{x};\theta,\sigma\right)\right)^{r+1}}{\left(T_I^*\left(\underline{x};\theta,\sigma\right) + log\left(1+y_j^{\sigma}\right)\right)^{r+2}}; \; y_j > 0. \tag{10}$$

Let us assume the transformation

$$Z = 1 - \left(\frac{T_I^*\left(\underline{x};\theta,\sigma\right)}{T_I^*\left(\underline{x};\theta,\sigma\right) + log\left(1+y_j^{\sigma}\right)}\right)^{r+1}$$

then the probability density function for the j^{th} ordered future observation becomes

$$\Phi_I\left(Z\right) = j\left({}^kC_j\right)\left(Z\right)^{j-1}\left(1-Z\right)^{k-j}; Z > 0. \tag{11}$$

Now, we say that (l_1, l_2) is a $100(1-\epsilon)\%$ prediction limits for a future random variable Y, if

$$Pr\left(l_1 \leq Y \leq l_2\right) = 1 - \epsilon. \tag{12}$$

Here l_1 and l_2 be the lower and upper Bayes prediction limits of the random variable Y, and $1-\epsilon$ is called the confidence prediction coefficient. To find the prediction limits under the two-sample plan for Y_j, j^{th} observation from a set of k future observations, we rewrite the Eq. (12) under the equal tail limits, as

$$Pr\left(Y_j \leq l_{1j}\right) = \frac{\epsilon}{2} = Pr\left(Y_j \leq l_{2j}\right) \forall\, j = 1, 2, ..., k. \tag{13}$$

Using the Eq. (11) and Eq. (13), the expressions of the limits for the j^{th} future observation are obtained by solving following equations

$$j\left({}^kC_j\right)\int_0^{\hat{l}_1} Z^{j-1}\left(1-Z\right)^{k-j} dZ = \frac{\epsilon}{2}$$

and

$$j\left({}^kC_j\right)\int_0^{\hat{l}_2} Z^{j-1}\left(1-Z\right)^{k-j} dZ = 1 - \frac{\epsilon}{2}, \tag{14}$$

where $\hat{l}_i = 1 - \left(\frac{T_I^*(\underline{x};\theta,\sigma)}{T_I^*(\underline{x};\theta,\sigma)+log\left(1+l_{ij}^{\sigma}\right)} \right)^{r+1}$; $i = 1, 2$.

Solving Eq. (14) for $j = 1$, the lower and upper Bayes prediction limits for the first future observation are given as

$$l_{11I} = \left\{ exp\left(\left(\epsilon^* - 1 \right) T_I^*\left(\underline{x};\theta,\sigma \right) \right) - 1 \right\}^{1/\sigma} \; ; \; \epsilon^* = \left(\frac{2-\epsilon}{2} \right)^{-1/k(r+1)}$$

and

$$l_{21I} = \left\{ exp\left(\left(\epsilon^{**} - 1 \right) T_I^*\left(\underline{x};\theta,\sigma \right) \right) - 1 \right\}^{1/\sigma} \; ; \; \epsilon^{**} = \left(\frac{\epsilon}{2} \right)^{-1/k(r+1)}.$$

Similarly, solving the Eq. (14) for $j = k$, the prediction limits for the last future observation is

$$l_{1kI} = \left\{ exp\left(\left(\tau^* - 1 \right) T_I^*\left(\underline{x};\theta,\sigma \right) \right) - 1 \right\}^{1/\sigma} \; ; \; \tau^* = \left(1 - \left(\frac{\epsilon}{2} \right)^{\frac{1}{k}} \right)^{-1/(r+1)}$$

and

$$l_{2kI} = \left\{ exp\left(\left(\tau^{**} - 1 \right) T_I^*\left(\underline{x};\theta,\sigma \right) \right) - 1 \right\}^{1/\sigma} \; ; \; \tau^{**} = \left(1 - \left(\frac{2-\epsilon}{2} \right)^{\frac{1}{k}} \right)^{-1/(r+1)}.$$

Hence, the Bayes prediction lengths for the smallest (first) and the largest (last) future observations are obtained as

$$L_{(IS)} = l_{21I} - l_{11I}$$

and

$$L_{(IL)} = l_{2kI} - l_{1kI}. \tag{15}$$

2.2. Right item-failure censoring

Since all n items from the considered model are put under the life test without replacement. In which only $r(\leq n)$ ordered items are measurable, while the remaining $(n-r)$ items are censored. These $(n-r)$ censored lifetimes will be ordered distinctly. This process is known as the right Item failure-censoring scheme (Prakash (2014)).

Now, let us consider a sequence of independent random sample from Burr Type-XII distribution of size n such as $x_{(1)}, x_{(2)}, ..., x_{(r-1)}, x_{(r)}, x_{(r+1)}, ..., x_{(n)}$. All n items are put to test without replacement and the first r items $\underline{x} \cong \left(x_{(1)}, x_{(2)}, ..., x_{(r-1)}, x_{(r)} \right)$ are fully measured while remaining $(n-r)$ items $\left(x_{(r+1)}, x_{(r+2)}, ..., x_{(n)} \right)$ are censored. Based on above the joint probability density function of these order statistics is defined as

$$f_R\left(\underline{x}|\theta,\sigma \right) \propto \left(\prod_{i=1}^{r} f\left(x_{(i)};\theta,\sigma \right) \right) \cdot \left(\prod_{i=r+1}^{n} \left(1 - F\left(x_{(i)};\theta,\sigma \right) \right) \right)$$

$$\Rightarrow f_R\left(\underline{x}|\theta,\sigma \right) \propto \theta^r exp\left(-\theta T_R\left(\underline{x};\theta,\sigma \right) \right) \; ; \; T_R\left(\underline{x};\theta,\sigma \right) = \sum_{i=1}^{n} log\left(1 + x_{(i)}^{\sigma} \right). \tag{16}$$

Using Eq. (5) and Eq. (16) in Eq. (6), the posterior density for unknown parameter θ under right item-failure censoring is obtained as

$$\pi_{R\theta}^* = \frac{\left(T_R^*\left(\underline{x};\theta,\sigma \right) \right)^{r+1}}{\Gamma(r+1)} \theta^r exp\left(-\theta T_R^*\left(\underline{x};\theta,\sigma \right) \right) \; ; \; T_R^*\left(\underline{x};\theta,\sigma \right) = T_R\left(\underline{x};\theta,\sigma \right) + \alpha. \tag{17}$$

On similar lines, the Bayes predictive density, cumulative predictive density functions of future observation Y and probability density function of the j^{th} ordered future observation are obtained respectively as

$$h_R\left(Y|\underline{x}\right) = (r+1)\sigma y^{\sigma-1}\left(1+y^{\sigma}\right)^{-1}\frac{\left(T_R^*\left(\underline{x};\theta,\sigma\right)\right)^{r+1}}{\left(T_R^*\left(\underline{x};\theta,\sigma\right)+log\left(1+y^{\sigma}\right)\right)^{r+2}},$$

$$G_R\left(Y|\underline{x}\right) = 1-\left(\frac{T_R^*\left(\underline{x};\theta,\sigma\right)}{T_R^*\left(\underline{x};\theta,\sigma\right)+log\left(1+y^{\sigma}\right)}\right)^{r+1}$$

and

$$\Phi_R\left(Z\right) = j\left({}^kC_j\right)\left(Z\right)^{j-1}\left(1-Z\right)^{k-j};Z>0 \tag{18}$$

where $Z = 1-\left(\frac{T_R^*\left(\underline{x};\theta,\sigma\right)}{T_R^*\left(\underline{x};\theta,\sigma\right)+log\left(1+y_j^{\sigma}\right)}\right)^{r+1}$.

Solving Eq. (18) for $j=1$ and $j=k$, the lower and upper Bayes prediction bound limits for first and last future observation are given respectively as

$$l_{11R} = \left\{exp\left(\left(\epsilon^*-1\right)T_R^*\left(\underline{x};\theta,\sigma\right)\right)-1\right\}^{1/\sigma},$$

$$l_{21R} = \left\{exp\left(\left(\epsilon^{**}-1\right)T_R^*\left(\underline{x};\theta,\sigma\right)\right)-1\right\}^{1/\sigma},$$

$$l_{1kR} = \left\{exp\left(\left(\tau^*-1\right)T_R^*\left(\underline{x};\theta,\sigma\right)\right)-1\right\}^{1/\sigma}$$

and

$$l_{2kR} = \left\{exp\left(\left(\tau^{**}-1\right)T_R^*\left(\underline{x};\theta,\sigma\right)\right)-1\right\}^{1/\sigma}.$$

Now, the Bayes prediction intervals for first and last future observations are obtained similarly as

$$L_{(RS)} = l_{21R}-l_{11R}$$

and

$$L_{(RL)} = l_{2kR}-l_{1kR}. \tag{19}$$

2.3. Progressive Type-II censoring

The progressive censoring seems to be a great importance in strategic interval experiments. In many industrial experiments involving lifetimes of machines or units, it is required to dismiss the experiments early with failures must be limited for various reasons. This censoring criterion plays a significant role in such lifetime studies, in which the experiments terminate early.

Let us suppose an experiment in which n independent and identical units $x_{(1)}, x_{(2)}, ..., x_{(n)}$ are placed on a live test at beginning time and first $r;(1\le r\le n)$ failure items are observed. At the time of each failure occurring prior to termination point, one (or more) enduring units detached from the test. The experiment is terminated at the time of r^{th} failure, and all remaining surviving units are removed from the test. See Prakash (2015) for more details on Progressive censoring.

Let $\underline{x}\cong\left(x_{(1)}, x_{(2)}, ..., x_{(r)}\right)$ are the lifetimes of completely observed units to fail and $R_1, R_2, ..., R_r$ are the numbers of units withdrawn at these failure times. Here, $R_1, R_2, ..., R_r$ all are predefined integers following the relation

$$\sum_{i=1}^{r}R_i+r=n.$$

Based on progressively type-ii censoring scheme the joint probability density function of order statistics $\underline{x} \cong \left(x_{(1)}, x_{(2)}, ..., x_{(r)}\right)$ is defined as

$$f_p\left(\underline{x}|\theta,\sigma\right) = C_p \prod_{i=1}^{r} f\left(x_{(i)};\theta,\sigma\right)\left(1 - F\left(x_{(i)};\theta,\sigma\right)\right)^{R_i}; \tag{20}$$

Here, C_p is known as progressive normalizing constant. Simplifying Eq. (20), we get

$$\Rightarrow f_P\left(\underline{x}|\theta,\sigma\right) \propto \theta^r exp\left(-\theta T_P\left(\underline{x};\theta,\sigma\right)\right)\ ;\ T_P\left(\underline{x};\theta,\sigma\right) = \sum_{i=1}^{r}\left(1+R_i\right) log\left(1+x_{(i)}^{\sigma}\right).$$

The posterior density about the parameter θ under progressive censoring plan is

$$\pi^*_{P\theta} = \frac{\left(T^*_P\left(\underline{x};\theta,\sigma\right)\right)^{r+1}}{\Gamma(r+1)}\theta^r exp\left(-\theta T^*_P\left(\underline{x};\theta,\sigma\right)\right)\ ;\ T^*_P\left(\underline{x};\theta,\sigma\right) = T_P\left(\underline{x};\theta,\sigma\right) + \alpha.$$

Similarly, the Bayes predictive density, cumulative predictive density functions of future observation Y and probability density function of the j^{th} ordered future observation under progressive censoring are obtained and given respectively as

$$h_P\left(Y|\underline{x}\right) = (r+1)\sigma y^{\sigma-1}\left(1+y^{\sigma}\right)^{-1}\frac{\left(T^*_P\left(\underline{x};\theta,\sigma\right)\right)^{r+1}}{\left(T^*_P\left(\underline{x};\theta,\sigma\right) + log\left(1+y^{\sigma}\right)\right)^{r+2}},$$

$$G_P\left(Y|\underline{x}\right) = 1 - \left(\frac{T^*_P\left(\underline{x};\theta,\sigma\right)}{T^*_P\left(\underline{x};\theta,\sigma\right) + log\left(1+y^{\sigma}\right)}\right)^{r+1}$$

and

$$\Phi_P\left(Z\right) = j\left({}^kC_j\right)\left(Z\right)^{j-1}\left(1-Z\right)^{k-j}; Z > 0 \tag{21}$$

where $Z = 1 - \left(\frac{T^*_P(\underline{x};\theta,\sigma)}{T^*_P(\underline{x};\theta,\sigma)+log\left(1+y_j^{\sigma}\right)}\right)^{r+1}$.

Substituting $j = 1$ and $j = k$ in Eq. (21). The lower and upper Bayes prediction bound limits for first and last future observation are given as

$$l_{11P} = \left\{exp\left(\left(\epsilon^* - 1\right)T^*_P\left(\underline{x};\theta,\sigma\right)\right) - 1\right\}^{1/\sigma},$$

$$l_{21P} = \left\{exp\left(\left(\epsilon^{**} - 1\right)T^*_P\left(\underline{x};\theta,\sigma\right)\right) - 1\right\}^{1/\sigma},$$

$$l_{1kP} = \left\{exp\left(\left(\tau^* - 1\right)T^*_P\left(\underline{x};\theta,\sigma\right)\right) - 1\right\}^{1/\sigma}.$$

and

$$l_{1kP} = \left\{exp\left(\left(\tau^{**} - 1\right)T^*_P\left(\underline{x};\theta,\sigma\right)\right) - 1\right\}^{1/\sigma}.$$

Thus, the Bayes prediction intervals for the smallest and the largest future observation are obtained and given as

$$L_{(PS)} = l_{21P} - l_{11P}$$

and

$$L_{(PL)} = l_{2kP} - l_{1kP}.$$

3. Numerical analysis

The performance of the proposed procedures is studied by a numerical illustration based on a real data set for a clinical trial describe a relief time (in hours) for 30 arthritic patients considered here form data provided by Wingo (1993) and used recently by Wu, Wu, Chen, Yu, and Lin (2010). The data are given in the Table (1).

Table 1: Relief time (in hours) for 30 arthritic patients

0.70	0.58	0.54	0.59	0.71	0.55	0.63	0.84	0.49	0.87
0.73	0.72	0.62	0.82	0.84	0.29	0.51	0.61	0.57	0.29
0.36	0.46	0.68	0.34	0.44	0.75	0.39	0.41	0.46	0.66

We fit the Burr Type-XII distribution to the given data in Table (1). The Kolmogorov-Smirnov (K-S) distances between the fitted and the empirical distribution functions is 0.0675 with p-value is > 0.05. Based on the K-S test statistic, Burr Type-XII distribution provides an adequate fit the data sets. In addition, the graph for both the empirical survival function and the estimated survival functions is given in Figure (3.3). (El-Sagheer (2015))

We carry out this comparison by considering the given data of size $n(= 30)$ with $\sigma(= 1.00)$ and $\alpha(= 0.50)$. The selected values of level of significance are $\epsilon = 99\%, 95\%, 90\%$.

3.1. Item-failure censoring scheme

Let the test is terminated when $r(= 5, 10, 15)$, as it is supposed from $n = 30$. Help of a considered set of parametric values, obtains the one-sided two-sample Bayes prediction bound lengths with the data given in Table (1) and presented in Table (3).

It is noted that when confidence level ϵ increases the length of intervals tends to be wider. A decreasing trend has been seen in bound lengths when censored sample size increases.

3.2. Right item-failure censoring scheme

The one-sided two-sample Bayes prediction bound lengths have been obtained under similar set of considered parametric set of values as discussed above and presented in Table (3) for right item-failure censoring data.

All properties have seen similar for the bound lengths obtained under item-failure censoring criterion. However, the bound lengths become narrower as compared to item-failure censoring criterion for all considered parametric set of values

3.3. Progressive censoring scheme

The Bayes prediction bound length under two-sample criterion have been obtained and presented in Table (3) for a similar set of parametric values as discussed above in censoring plan $R_i; i = 1, 2, ..., r$, given in (2).

Again, all the behaviors have seen similar as discussed above when compared with both censoring criteria. Further, it is noted that the magnitude of bound lengths under progressive censoring criteria are wider than compared to item-failure or right item-failure censoring criterion. It is also remarkable that for small confidence level, the bound length for largest observation is narrower as compared to the item-failure censoring criterion.

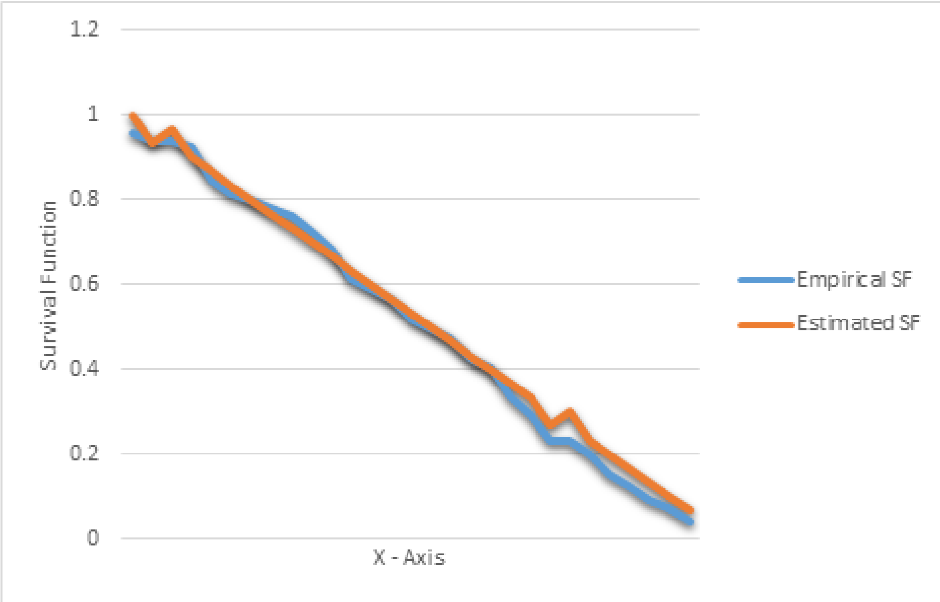

Figure 1: Empirical and estimated survival functions

Table 2: Different progressive censoring plan

Case	m	$R_i; i = 1, 2, ..., m$
1	5	1 2 1 0 1
2	10	1 0 0 3 0 0 1 0 0 1
3	15	1 0 2 0 0 1 0 2 0 0 0 1 0 0 1

4. Simulation study

Based on simulation, the performances of the procedures are studied in the present section.

Using Eq. (5), the values of shape parameter θ have been generated by using $\alpha(= 0.25, 0.50, 1.00)$. Using these three generated values of θ with a known set of values of parameter $\sigma(= 0.50, 1.00, 2.00)$, generates $10,000$ random samples, each of size $n = 30$.

All desired censored samples are generated by using following relation $x_i = \left\{(1 - U_i)^{-\frac{1}{\theta}} - 1\right\}^{\frac{1}{\sigma}}$. Here, U_i are independently distributed $U(0, 1)$. The one-sided two-sample Bayes prediction bound lengths based on simulated data are presented in the Tables 04-06 for item-failure, right item-failure, and progressive censored data respectively.

The bound length becomes wider as combination of prior parameter increase. However, a decreasing trend has seen for higher set of prior values ($\alpha = 1.00, \sigma = 2.00$). All other properties have seen similar as discussed in the previous section.

Conclusion

The properties of Bayes prediction bound lengths based on two-sample technique are the main aim of the present discussion. The underlying model is assumed here as the Burr Type-XII distribution and the analysis presented by simulated data set and a real data set provided by Wingo (1993). The item-failure, right item-failure, and progressive Type-II censoring is used for the present comparative study.

Table 3: Two-sample Bayes prediction bound lengths under different censoring plans

$\alpha = 0.50$	Item-Failure Censoring Plan					
$\sigma = 1.00$	The First Future Observation			The Last Future Observation		
$r \downarrow \epsilon \rightarrow$	99%	95%	90%	99%	95%	90%
5	3.3742	3.3501	3.3237	4.7201	4.6645	4.5772
10	2.5178	2.4556	2.3996	3.3748	3.3128	3.1849
15	2.1914	1.9791	1.9152	2.8201	2.6846	2.5436
	Right Item-Failure Censoring Plan					
5	3.2511	3.2078	3.1423	4.5177	4.2942	4.1967
10	2.4158	2.3359	2.2312	3.2116	3.1018	3.0186
15	2.0414	1.9268	1.8753	2.6171	2.5366	2.4507
	Progressive Type-II Censoring Plan					
5	3.8061	3.7488	3.6002	5.1837	4.8177	3.9006
10	3.1998	3.0739	2.8952	3.5538	3.1084	2.9082
15	2.7939	2.5121	2.4109	3.1664	3.0008	2.4610

Table 4: Bound lengths under item-failure censoring plan

$n = 30$		The First Future Observation			The Last Future Observation		
$(\alpha, \sigma) \downarrow$	$r \downarrow \epsilon \rightarrow$	99%	95%	90%	99%	95%	90%
	5	2.9547	2.9031	2.6749	3.0745	3.0616	3.0194
0.25, 0.50	10	2.1815	2.1215	2.0149	2.3124	2.1822	2.0974
	15	1.6953	1.4341	1.1327	1.9057	1.6216	1.5998
	5	3.1231	3.0178	2.9104	3.3538	3.3297	3.2607
0.50, 1.00	10	2.3081	2.0387	1.8196	2.5024	2.4504	2.2652
	15	1.9128	1.8114	1.6418	2.1488	1.9871	1.9333
	5	3.0445	3.0193	2.9522	3.3124	3.2786	3.0501
1.00, 2.00	10	2.2696	2.1615	2.1527	2.4615	2.4101	2.2336
	15	1.8892	1.7889	1.7203	2.1235	1.9426	1.9294

Based on selected parametric values, the one-sided two-sample Bayes prediction bound lengths are wider under the Progressive censoring scheme as compared to other censoring patterns. It is also remarkable that for small confidence level, the bound length for largest observation is narrower under Progressive censoring criterion as compared to the item-failure censoring criterion.

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Table 5: Bound lengths under right item-failure censoring plan

$n = 30$		The First Future Observation			The Last Future Observation		
$(\alpha, \sigma) \downarrow$	$r \downarrow \epsilon \rightarrow$	99%	95%	90%	99%	95%	90%
0.25, 0.50	5	2.7179	2.7089	2.3797	3.0415	3.0316	3.0104
	10	2.1152	1.9841	1.8189	2.2224	2.1212	2.0714
	15	1.6138	1.3508	1.1612	1.8570	1.5206	1.4778
0.50, 1.00	5	3.0320	2.8185	2.3913	3.3438	3.2297	3.2107
	10	2.1233	1.9133	1.6351	2.3524	2.2314	2.1782
	15	1.7911	1.5373	1.2124	2.1187	1.9711	1.8333
1.00, 2.00	5	2.9029	2.8089	2.4484	3.2124	3.1860	3.0310
	10	2.0641	1.9005	1.7893	2.3315	2.1401	2.0836
	15	1.7804	1.3134	1.2128	2.0925	1.9006	1.7294

Table 6: Bound lengths under progressive type-II censoring plan

$n = 30$		The First Future Observation			The Last Future Observation		
$(\alpha, \sigma) \downarrow$	$r \downarrow \epsilon \rightarrow$	99%	95%	90%	99%	95%	90%
0.25, 0.50	5	3.2003	2.9086	2.6928	3.3415	3.3165	3.2606
	10	2.3002	2.2605	2.1892	2.4124	2.3305	2.1975
	15	1.7975	1.7801	1.6168	2.1306	1.9155	1.6792
0.50, 1.00	5	3.3233	3.0875	2.9622	3.6139	3.5078	3.2754
	10	2.5178	2.0889	1.9303	2.7289	2.6124	2.2698
	15	2.1783	1.8681	1.7838	2.3047	2.1159	2.0706
1.00, 2.00	5	3.3079	3.2105	3.0276	3.4599	3.2622	3.0314
	10	2.4659	2.1485	1.9389	2.6144	2.3186	2.1355
	15	2.0126	1.9037	1.4691	2.1072	2.0107	1.9163

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