Austrian Journal of Statistics
July 2022, Volume 51, 96–119.
http://www.ajs.or.at/
doi:10.17713/ajs.v51i4.1170

A Comparative Analysis of Semiparametric Tests for Fractional Cointegration in Panel Data Models

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Abstract

Several authors have studied fractional cointegration in time series data, but little or no consideration has been extended to panel data settings. Therefore, in this paper, we compare the finite sample behaviour of existing fractional cointegration time-series test procedures in panel data settings. This comparison is performed to determine the best tests that can be adapted to fractional cointegration in panel data settings. Specifically, simulation studies and real-life data analysis were performed to study the changes in the empirical type I error rate and power of six semiparametric fractional cointegration tests in panel settings. The various results revealed the limitations of the tests in the nonstationary and low or high correlation of the residual errors conditions. Also, two of the test procedures were recommended for testing the null hypothesis of no fractional cointegration in both time series and panel data settings.

Keywords: panel data models, long memory, fractional cointegration, semiparametric tests.

1. Introduction

In econometrics, whether a data is experimental or observational, it can be majorly classified into cross-sectional data, time-series data and panel data whereas the panel data combines the attributes of both the cross-sectional data and the time series. However, in macroeconomics and finance, variables are usually presented in panels to describe the varying characteristics of the diffrent entities such as currencies, asset, countries, income, people, and so on. Since panel data allows for interactions of cross-sections with each other, it leads to a more robust inference when correctly specified.

Cointegration techniques have been widely used for decades in empirical studies, it naturally occurs in economics and finance, and it allows modelling of equilibrium relationships between non-stationary time series while fractional cointegration is well known to overcome the short-comings of cointegration by allowing for non-integer integration orders of the variables in the system and any possible non-zero memory order in the cointegrating residuals as long as it is reduced in comparison to the original system Johansen and Nielsen (2012).

Studying fractional cointegration has led to the development of various test and estimation procedures to determine its presence in a multivariate time series. Some of the tests used include Marmol and Velasco (2004), Chen and Hurvich (2006), Johansen (2008), Robinson

(2008a), Nielsen (2010), Avarucci and Velasco (2009), Łasak (2010), Johansen and Nielsen (2012), Johansen and Nielsen (2019) and Souza, Reisen, Franco, and Bondon (2018) among others.

In a recent Monte-Carlo based comparative study of existing fractional cointegration techniques by Leschinski, Voges, and Sibbertsen (2020), it was observed that some of the existing fractional cointegration test procedures exhibit various weaknesses in their finite sample behaviour. Specifically, some tests show low power or size bias in the presence of correlated short-run components. This concerns mostly the methods of Nielsen and Shimotsu (2007) (or Robinson and Yajima (2002)), Marmol and Velasco (2004), and Hualde and Velasco (2008).

Leschinski *et al.* (2020) also find that the size properties of the tests in the triangular case and the common-components model is generally comparable. For the power of the tests, however, there are important differences between the two cases. In particular, the test of Chen and Hurvich (2006) has much better power for stationary systems under the common components specification. In contrast, the methods of Robinson and Yajima (2002) and Hualde and Velasco (2008) become worse in their ability to detect fractional cointegration.

Moreover, the difficulties in finding a long time series and the low power of the Augmented Dickey-Fuller and Dickey-Fuller unit root tests for the univariate case made researchers develop unit root and cointegration tests for panel data.

McCoskey and Kao (1998) derived a panel cointegration test for the null of cointegration which is an extension of the LM test and the locally best-unbiased invariant (LBUI) test for a Moving Average (MA) root and Kao, Chiang, and Chen (1999) considered the spurious regression for the panel data and introduced the Dickey-Fuller and Augmented Dickey-Fuller type tests. Kao *et al.* (1999) proposed four different DF type of test statistics and used the sequential limit theory of Phillips and Moon (1999) to derive the asymptotic distributions of these statistics.

However, Larsson, Lyhagen, and Löthgren (2001) suggested the panel cointegration test statistic based on cross-sectional independence. While Groen and Kleibergen (2003) on the other hand presented how homogenous and heterogeneous cointegration vectors are estimated within a maximum-likelihood framework using the GMM procedure. The authors also proposed a likelihood ratio test for the common cointegration rank, which is based on the GMM estimators and the cross-sectional dependence.

Of all the work done on fractional cointegration and panel data analysis, no one has considered the two scenarios together, i.e. fractional cointegration in panel data parlance. Still, a lot has been done on fractional cointegration in time series model. Therefore, in this paper, we compare the finite sample behaviour of existing semiparametric fractional cointegration time-series test procedures in panel data settings. This comparison is performed to determine the best tests that can be adapted to fractional cointegration in panel data settings. In this study, we focus on the changes in power and size of the panel fractional cointegration tests when time and cross-section dimensions and various parameters like the correlation parameter and bandwidth in the data generating process varies for each cross-section in homogenous and heterogeneous data structures. The semiparametric tests were considered in this paper as they are more popular than their counterparts.

2. Fractional cointegration

Suppose x_t and y_t are two processes that are both $I(d)$, Engle and Granger (1987) reported that for a certain scalar $a \neq 0$, a linear combination $w_t = y_t - ax_t$, will also be $I(d)$, with the possibility that w_t can be $I(d-b)$ with $b > 0$. Thus, given two real numbers d, b, the components of the vector c_t are said to be cointegrated of order d, b, denoted as $c_t \sim CI(d, b)$ if:

i all the components of c_t are $I(d)$, ii there exists a vector $\alpha \neq 0$ such that $s_t = \alpha' c_t \sim I(\gamma) = I(d-b), b > 0$,

where α and s_t are called the cointegration vector and error respectively Caporale and Gil-Alana (2014). A simple bivariate system of fractional cointegrated x_t and y_t processes can be defined as:

$$\begin{aligned} y_t &= \beta x_t + (1-L)^{-\gamma}\epsilon_{1t} \\ x_t &= (1-L)^{-d}\epsilon_{2t} \end{aligned} \tag{1}$$

for positive t. The vector $\epsilon_t = (\epsilon_{1t}, \epsilon_{2t})'$ is now a bivariate zero mean covariance stationary $I(0)$ process, $\beta \neq 0$ and $\gamma < d$. In equation (1) x_t and y_t are both $I(d)$ and $\epsilon_{1t} = y_t - \beta x_t$ is $I(\gamma)$. The bivariate system in (1) can be reduced to classical cointegration of Phillips (1991) if $\gamma = 0$ and $d = 1$ which is denoted by $CI(1,1)$. The lag operator $(1-L)^{-d}$ is obtained using $(1-L)^{-d} = \frac{\Gamma(j+d)}{\Gamma(d)\Gamma(j+1)}$, where $\Gamma(z) = \int_0^\infty \omega^z \exp^{-\omega} d\omega$.

In contrast to standard $CI(1,1)$ cointegration, the memory parameter d is unknown in fractionally cointegrated systems and has to be estimated.

2.1. Fractional cointegration in panel models

In this section, we consider a panel model similar to (1) but with the introduction of $i = 1, 2, \ldots, N$ cross-sectional units. The general fractional cointegration panel model derived from (1) is given by:

$$\begin{aligned} y_{it} &= \beta x_{it} + (1-L)^{-\gamma}\epsilon_{1it} \\ x_{it} &= (1-L)^{-d}\epsilon_{2it} \end{aligned} \tag{2}$$

where the cointegration parameter β is assumed to be constant over i cross-sectional units. Model (2) is the simplistic form of fractional integration in panel settings and differs from the cross-sectional dependence one proposed by Ergemen and Velasco (2017). In this paper, we extend (2) to capture fixed effect homogenous (Pooled) and heterogenous panel data settings. The homogenous (Pooled) model is given by:

$$\begin{aligned} y_{it} &= \mu + \beta x_{it} + (1-L)^{-\gamma}\epsilon_{1it} \\ x_{it} &= (1-L)^{-d}\epsilon_{2it}, \end{aligned} \tag{3}$$

and the heterogenous model is given by;

$$\begin{aligned} y_{it} &= \mu_i + \beta x_{it} + (1-L)^{-\gamma}\epsilon_{1it} \\ x_{it} &= (1-L)^{-d}\epsilon_{2it}. \end{aligned} \tag{4}$$

The parameter μ or μ_i is the fixed effect coefficient for the pooled or ith cross-sectional units. Given models (3) and (4), the fractional cointegration tests in (1) can also be applied. The comparative analysis for semiparametric tests conducted in Leschinski *et al.* (2020) holds for (1). The focus of this paper is to determine the validity of some of these tests in panel data settings captured in (3) and (4).

3. Fractional cointegration tests

In this section, we present a brief review of six fractional cointegration tests that are commonly used to test the existence of fractional cointegration in time series data. These tests are broadly classified into spectral-based and residual-based tests Leschinski *et al.* (2020). The relevant hypotheses to test whether the two processes are fractionally cointegrated are:

$H_0 : x_t$ and y_t are not fractionally cointegrated $(d = \gamma)$,
$H_1 : x_t$ and y_t are fractionally cointegrated $(d > \gamma)$.

3.1. Spectral-based fractional cointegration tests

The spectral density of a $p-$dimensional long memory vector X_t can be defined as;

$$f_X(\lambda) \sim \Lambda_j(d) G \overline{\Lambda_j(d)}, \quad \lambda \to 0^+, \tag{5}$$

where $\Lambda_j(d) = diag\big(\lambda^{-d_1} e^{i\pi d_1/2}, \ldots, \lambda^{-d_p} e^{i\pi d_p/2}\big)$ is a $p \times p$ diagonal matrix, $\overline{\Lambda_j(d)}$ is its complex conjugate, and G is a real, symmetric and positive definite matrix.

The spectral-based test procedures utilize the zero frequency property of rescaled spectral matrix G in (5) to determine the fractional cointegrating rank r of a $p-$dimensional time series X_t. The matrix G has a reduced rank if and only if X_t is fractionally cointegrated. If fractional cointegration is present, the number of eigenvalues that are equal to zero corresponds to the cointegrating rank r and therefore to the number of cointegrating relationships Leschinski *et al.* (2020).

Robinson (2008) test

Robinson (2008a) proposed a test statistic that is based on the objective function of the multivariate local Whittle estimator of Nielsen and Shimotsu (2007) given by:

$$S(d) = \log\det \hat{G}^*(d) - \frac{2pd}{m}\sum_{j=1}^{m}\log\lambda_j \tag{6}$$

where $\hat{G}^*(d) = \frac{1}{m}\sum_{j=1}^{m} I_X(\lambda_j)\lambda_j^{2d}$, $I_X(\lambda_j) = w_x(\lambda_j)\overline{w_x(\lambda_j)}$ is the periodogram of $p-$dimensional series X_t at the Fourier frequencies $\lambda_j = 2\pi j/T$, $j = 1, 2, \ldots \lfloor T/2 \rfloor$, $\lfloor . \rfloor$ denotes the greatest integer smaller than the argument, $w_x(\lambda_j) = \frac{1}{\sqrt{2\pi T}}\sum_{t=1}^{T} X_t e^{i\lambda t}$ and m is the bandwidth.

Robinson (2008a) used the derivative of equation (6) given by:

$$s^*(d) = tr\Big(\hat{G}^*(d)^{-1}\hat{H}^*(d)\Big) \tag{7}$$

where $\hat{H}^*(d) = \frac{1}{m}\sum_{j=1}^{m} v_j I_X(\lambda_j)\lambda_j^{2d}$ and $v_j = \log j \sum_{k=1}^{m} k$ to derive the test statistic

$$X^*_{Rob} = \frac{m s^*\big(\hat{d}_{LW}\big)^2}{p^2 tr\big(\hat{F}^{*2}\big) - p} \tag{8}$$

with $\hat{d}_{LW}$ being the pooled local Whittle estimates of the memory parameter d for each component series (x_t, y_t) and $\hat{F}^* = \hat{R}^{*^{-1/2}}\hat{G}^*\big(\hat{d}_{LW}\big)\hat{R}^{*^{-1/2}}$, $\hat{R}^* = diag(\hat{g}^*_{11}, \ldots, \hat{g}^*_{pp})$, $\hat{g}^*_{aa}$, $a = 1, \ldots, p$ are the diagonal elements of $\hat{G}^*\big(\hat{d}_{LW}\big)$. Under H_0, the statistic $X^*_{Rob} \overset{D}{\sim} \chi^2_1$. The test can be applied to series with dimension greater than 2 but restricted the memory parameter to $d \in (-1/2, 1/2)$ and specifically $d \in (0, 1/2)$. Thus, the test is only applicable to stationary process.

Souza et al. (2018) test

The Souza *et al.* (2018) test is based on the estimate of memory reduction parameter b obtained from the determinant of trimmed and truncated spectral matrix of the fractionally differenced process using log-periodogram regression.

Suppose we let $\Delta^d X_t = (\Delta^d y_t, \Delta^d x_t)$ be a fractionally differenced process with spectral density matrix $f_{\Delta^d}(\lambda)$ then the determinant $D_{\Delta^d}(\lambda) \sim \tilde{g}|1-e^{-i\lambda}|^{2b}$, (where $\tilde{g}$ is a constant) of $f_{\Delta^d}(\lambda)$ depends on the memory reduction parameter $b \in (0, d)$. The parameter b is obtained from $\log(D_{\Delta^d}(\lambda))$ via a log-periodogram regression,

$$\log(D_{\Delta^d}(\lambda)) \sim \log\tilde{g} + 2b\log|1-e^{-i\lambda}| + \log\left(\frac{\tilde{g}^*(\lambda)}{\tilde{g}}\right) \tag{9}$$

where $\tilde{g}^*(\lambda) = \tilde{g}$, when $\lambda \to 0^+$.

A feasible estimate of b is obtained by estimating $f_{\Delta^d}(\lambda)$ at Fourier frequencies $j = l, l+(2l-1)+l+2(2l-1), \ldots, m(2l-1), m$ with $l < m < T$. The estimate of the spectral density is;

$$\hat{f}_{\Delta^d}(\lambda) = \frac{1}{2l-1} \sum_{k=j-(l-1)}^{j+(l-1)} I_{\Delta^d}(\lambda_k),$$

where $I_{\Delta^d}(\lambda_k)$ is the periodogram of the fractionally differenced series. For every j, $\hat{f}_{\Delta^d}(\lambda)$ is estimated using the local average of the periodogram at frequency j and the $l+1$ frequencies to its left and right and the j are spaced so that the local averages are non-overlapping. Consequently, the estimator of b is given by

$$\hat{b} = \left(\sum_{j=l+1}^{m} \tilde{Z}_j^{*2} \right)^{-1} \sum_{j=l+1}^{m} \tilde{Z}_j^* \log \hat{D}_{\Delta}^d(\lambda_j), \tag{10}$$

where $\tilde{Z}_j^* = Z_j^* - \bar{Z}^*$, $Z_j^* = \log|1 - e^{i\lambda}| = \log(2 - 2\cos(\lambda_j))$, and $\bar{Z}^*$ is the mean of Z_j^*. The relevant hypotheses here are now;

$H_0 : x_t$ and y_t are not fractionally cointegrated $(b = 0)$,
$H_1 : x_t$ and y_t are fractionally cointegrated $(b > 0)$.

Under H_0, it can be assumed that l and m satisfy the condition $\frac{l+1}{m} + \frac{m}{T} + \frac{1}{m} + \frac{\log m}{m} \to 0$ as $T \to \infty$ and thus $\hat{b}$ is consistent and asymptotically normal with variance $\sigma_b^2 = \frac{1}{m}(\Psi^{(1)}(2l+1) + \Psi^{(1)}(2l))$, $\Psi^{(1)} = \frac{\delta^2 \log \Gamma(x)}{\delta x^2}$ is the first order polygamma function. Correspondingly, Souza *et al.* (2018) proposed the wald t-test statistic:

$$t_{SRFB} = \frac{\hat{b}}{\sqrt{\sigma_b^2}} \overset{D}{\sim} N(0,1). \tag{11}$$

The $t-$test statistic in equation (11) is unrestricted like Robinson (2008a) as it is applicable to any values of d and b. However, it is only applicabe to bivariate process Leschinski *et al.* (2020).

3.2. Residual-based fractional cointegration tests

Suppose we define $w_t = y_t - \beta x_t$ to be fractionally cointegrated $I(\gamma)$, we can test that the components x_t and y_t are fractionally cointegrated if $\gamma < d$ based on the estimate of the residual $\hat{w}_t$. The Ordinary Least Square (OLS) method is commonly used to estimate the cointegration relationship β. The estimate is consistent under the alternaive hypothesis $(\gamma < d)$ provided $d > 0.5$ but inconsistent under the null hypothesis $(\gamma = d)$.

Marmol and Velasco (2004) test

Marmol and Velasco (2004) proposed the Narrow Band estimator $\hat{\beta}^{NB}$ that is consistent under H_0 but inconsitent under H_1. Marmol and Velasco (2004) defined the $\hat{\beta}^{NB}$ as:

$$\hat{\beta}^{NB}(\hat{d}, \hat{\gamma}) = \hat{G}_{xx}^{MV}(\hat{d})^{-1} \hat{g}_{xy}^{MV}(\hat{\gamma}), \tag{12}$$

with $\hat{G}_{xx}^{MV}(\hat{d}) = \frac{2\pi}{m} \sum_{j=1}^{m} \tilde{\Lambda}_j(d)^{-1} \Re\{I_{xx}(\lambda_j)\} \tilde{\Lambda}_j^{-1}(d)$, $\hat{g}_{xy}^{MV}(\hat{\gamma}) = \sum_{j=1}^{m} \Re\{I_{xy}(\lambda_j)\lambda_j^{2(\hat{\gamma}-1)}\}$, and $I_{xx}(\lambda_j)$, $I_{xy}(\lambda_j)$ are the elements of $I_{\Delta X \Delta X}(\lambda_j)$ which is the periodogram of the differenced series ΔX_t while $\tilde{\Lambda}_j(d) = diag(\lambda_j^{1-d}, \ldots, \lambda_j^{1-d})$. Marmol and Velasco (2004) compared the

β_{OLS} with $\hat{\beta}^{NB}$ using the normalizing variance $\hat{V}_{MV}^{-1}$ estimated from the periodogram of the OLS residuals $\hat{w}_t$ and x_t. The test statistic is given by

$$W_{MV} = \frac{1}{p-1}\left(\hat{\beta}_{OLS} - \hat{\beta}_{NB}\right)' \hat{V}_{MV}^{-1}\left(\hat{\beta}_{OLS} - \hat{\beta}_{NB}\right), \tag{13}$$

where $\hat{V}_{MV}^{-1} = \left(\sum_{j=-m}^{m} I_{xx}(\lambda_j)\right)^{-1} \sum_{j=-m}^{m} I_{xx}(\lambda_j) I_{\hat{w}\hat{w}}(\lambda_j) \left(\sum_{j=-m}^{m} I_{xx}(\lambda_j)\right)^{-1}$. The asymptotic distribution of W_{MV} was derived by Marmol and Velasco (2004) and critical values provided for dimensions up to $p = 5$.

Chen and Hurvich (2006) test

Chen and Hurvich (2006) proposed a more direct test that utilizes the residual $w_t = y_t - \beta x_t$. The residual is first multiplied by the eigenvectors $\chi_{a,I_X^{av}}$ of the averaged periodogram $I_X^{av}(\lambda_j)$. The resulting transformed first and last residuals $\hat{w}_{1t}^{av}$ and $\hat{w}_{pt}^{av}$ are used to estimate the memory parameters $\hat{d}$ and $\hat{\gamma}$. Chen and Hurvich (2006) presents the statistic

$$T_{CH} = \frac{\hat{d}_{w_1} - \hat{\gamma}_{w_p}}{\sqrt{V_{CH}/m}} \tag{14}$$

where $V_{CH} = 0.5\left(\frac{\Gamma(4h-3)\Gamma^4(h)}{\Gamma^4(2h-1)}\right)$, h is the apriori differencing parameter for X_t. Under H_0, $T_{CH} \overset{D}{\sim} N(0,1)$.

Nielsen (2010) test

Nielsen (2010) introduced an alternative approach for testing the null hypothesis of no fractional cointegration that is based on sequential testing. The appraoch utilizes the variance-ratio statistic with the assumption that the process X_t is $I(d)$ and the resultant residuals is $I(\gamma) < 1/2 < d$. The procedure start by first computing the centered version of X_t denoted by $C_t = X_t - \bar{X}_t$, where $\bar{X}_t$ is the mean vector of the process and its fractionally cointegrated version defined as $\tilde{C}_t = \Delta^{\epsilon} C_t$. Thus, Nielsen (2010) defined the variance-ratio as

$$VR_T = \sum_{t=1}^{T} C_t C_t' \left(\sum_{t=1}^{T} \tilde{C}_t \tilde{C}_t'\right)^{-1}. \tag{15}$$

Under H_1 the rank of VR_T reduce to $p - r$. The proposed non-parametric traced test by Nielsen (2010) is

$$C = T^{2\epsilon} \sum_{k=1}^{p-r} \hat{ev}_k, \qquad r = 1, 2, 3, \ldots, p-1, \tag{16}$$

with ev being the eigenvalues of VR_T and r is the number of cointegrating relationships under H_0. Equation (16) is used to test the hypothesis that the cointeration rank r equals r_0 under H_0 and greater than r_0 under H_1. The limiting distribution of X_{Neil} and corresponding critical values for varying d and dimension $p - r$ are provided in Nielsen (2010). The major drawback of Nielsen (2010) approach is that it assumes the process X_t is nonstationary.

Wang et al. (2015) test

The Wang, Wang, and Chan (2015) fractional cointegration test is based on second components x_t of the process $X_t = (y_t, x_t)$ and the associated residuals ϵ_{2t}. Wang *et al.* (2015) constructed a simple t-like test statistic that utilizes the spetral density of the component x_t

denoted as $\hat{f}_{22} = \frac{1}{2\pi T}\sum_{t=1}^{T}(\Delta^{\hat{d}} x_t)^2$ and the fractional cointegration parameter γ of residual ϵ_{2t}. Thus, the statistic is given as

$$F_{Wang} = \frac{\sum_{t=1}^{T}\Delta^{\hat{\gamma}} x_t}{\sqrt{2\pi T \hat{f}_{22}}} \overset{H_0}{\to} N(0,1). \tag{17}$$

The method does not imposed any restriction on the memory parameter b, but requires $d > 0.5$ so that the cointegrating vector β can be estimated using OLS.

4. Simulation study

The structure of the simulations to which results follows is divided into three phases: (i.) Control (Traditional Time series model); (ii.) Moderate (Homogenous fixed effect panel model) and (iii.) High (Heterogenous fixed effect panel model). Each scenario is replicated over two bandwidth power values $\eta_m = (0.65 \ \& \ 0.75)$, where bandwidth $m = T^{\eta_m}$ as required for each test procedure. In addition, for each scenario, 5000 replications and three different correlation values $\rho = (0, 0.4, \ \& \ 0.9)$ values were used. The empircal type I error rates and power for the six tests were calculated using the average of the 5000 runs of each simulated model and testing. Similar procedure was used in Jamil, Abdullah, Kek, Olaniran, and Amran (2017); Olaniran and Yahya (2017); Olaniran and Abdullah (2019b, 2020); Popoola, Yahya, Popoola, and Olaniran (2020); Olaniran and Abdullah (2019a). For brevity, each test is presented in tables and figures as authors' initial letters and the corresponding publication year.

4.1. Scenario 1 (control): simulation procedure for time series model

We tried to have a control model structure to which the test procedures were originally designed to accommodate. Here, we assume a typical time series structure with no cross-sectional units. That is T is the total sample size. The model is:

$$\begin{aligned} y_t &= \beta x_t + \Delta^{-\gamma}\epsilon_{1t} \\ x_t &= \Delta^{-d}\epsilon_{2t} \end{aligned} \tag{18}$$

where $\beta = -1$, $\epsilon_t = (\epsilon_{1t}, \epsilon_{2t})'$ being a Gaussian white noise with $E(\epsilon_t) = 0$, $Var(\epsilon_{1t}) = Var(\epsilon_{2t}) = 1$ and $Cov(\epsilon_{1t}, \epsilon_{2t}) = \rho$. We consider cases with $\rho = 0, 0.4 \ \& \ 0.9$ and sample sizes $T = 50, 100, 150, 200, 250, 300, 350, 400, 450, 500$ and bandwidths power $\eta_m = 0.65, 0.75$ separately. We set $d = 0.4, 0.8$ and $\gamma = 0.2, 0.4, 0.8$ for each d. The results for bandwidth power $\eta_m = 0.75$ were presented in Figure 1 - 2 while results for $\eta_m = 0.65$ are shown in Figure 3 - 4. For all tests, type I error was fixed at 5%.

Figure 1 presents the simulation results for the empirical type I error rate, which was used to assess the validity of the tests at the control level and bandwidth 0.75. A test is valid if its empirical type I error rate revolves around the fixed 5% value imposed. At $d = \gamma = 0.8$, the results observed indicate that the methods of Chen and Hurvich (2006), Marmol and Velasco (2004) and Wang *et al.* (2015) overestimates the imposed threshold implying they rejected the null hypothesis more than expected. In contrast, the method of Nielsen (2010) and Souza *et al.* (2018) underestimates the imposed threshold implying they rejected the null hypothesis less than expected. While we can say the methods of Chen and Hurvich (2006), Marmol and Velasco (2004) and Wang *et al.* (2015) are restrictive; we can say the methods of Nielsen (2010) and Souza *et al.* (2018) are conservative. However, it was found that the method of Robinson (2008a) are approximately exact (closer to the threshold), especially at smaller sample size. Also, the empirical type I error rates of methods of Chen and Hurvich (2006), Marmol and Velasco (2004), Nielsen (2010), Robinson (2008a), and Wang *et al.* (2015) consistently increases with increase in sample sizes while the empirical type I error rates of the method of Souza *et al.* (2018) consistently reduces with increase in sample sizes.

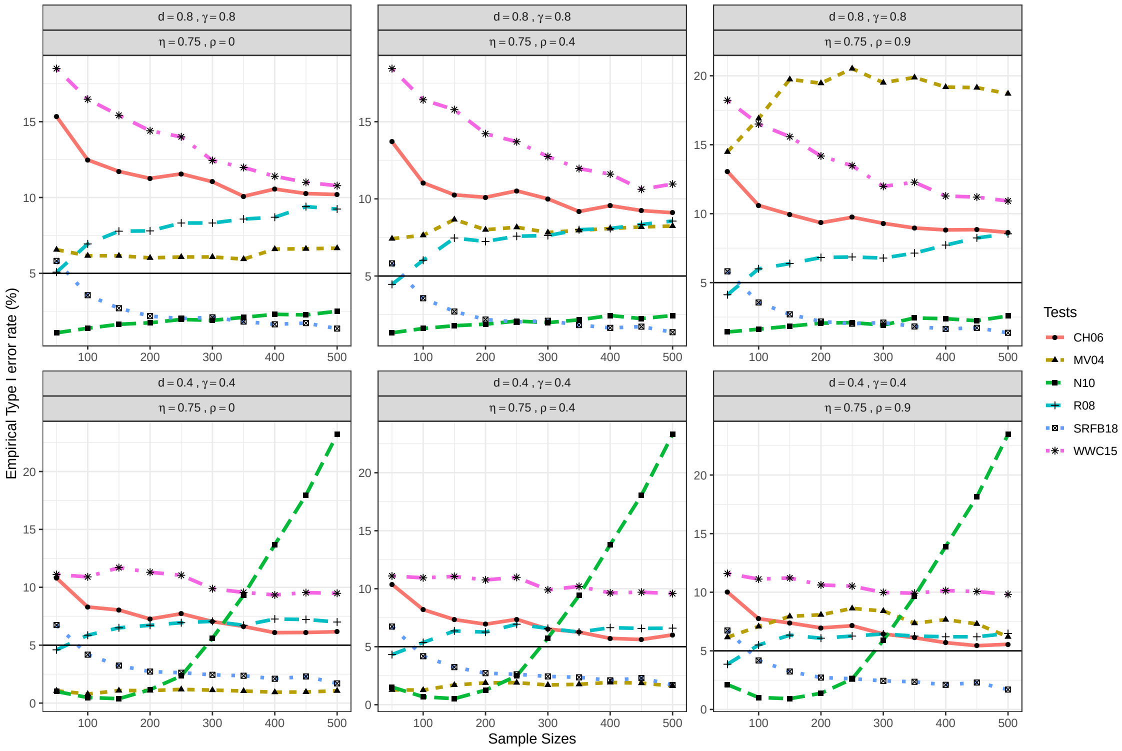

Figure 1: Empirical type I error rate (%) for time series model at bandwidth $\eta_m = 0.75$, $d = \gamma = 0.8$, $d = \gamma = 0.4$ and (i.) $\rho = 0$, (ii.) $\rho = 0.4$, (iii.) $\rho = 0.9$

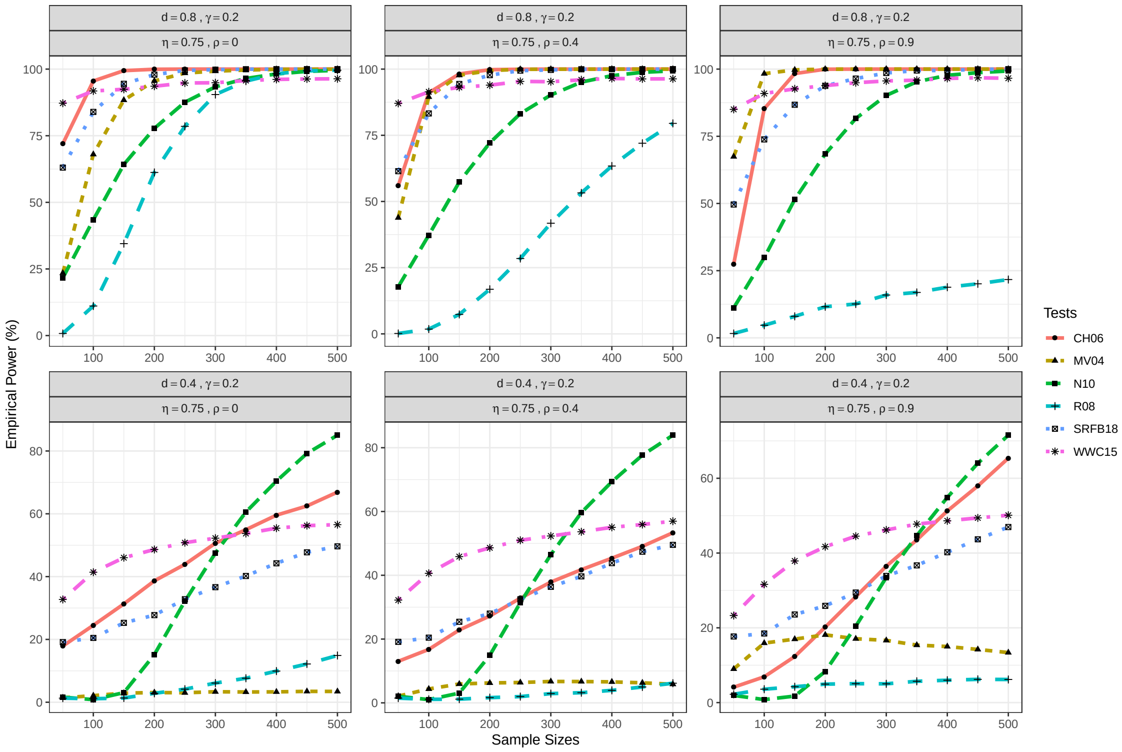

Figure 2: Empirical power (%) for time series model at bandwidth $\eta_m = 0.75$, $d = 0.8, \gamma = 0.2$, $d = 0.4, \gamma = 0.2$, and (i.) $\rho = 0$, (ii.) $\rho = 0.4$, (iii.) $\rho = 0.9$

Table 1: Average empirical type I error rate (when $d = \gamma = 0.8$) and power (when $d = 0.8$ and $\gamma = 0.2$) of time series model for the six tests at bandwidth $\eta_m = 0.65, 0.75$ and correlation $\rho = 0.0, 0.4, 0.9$

Type I error rate (%)						
Method	$\rho = 0.0$		$\rho = 0.4$		$\rho = 0.9$	
	0.65	0.75	0.65	0.75	0.65	0.75
CH06	1.536	11.450	10.270	10.270	0.238	9.724
MV04	4.674	6.288	8.016	8.016	21.318	18.750
N10	35.918	1.886	1.992	1.992	6.730	2.060
R08	7.670	8.016	7.336	7.336	5.204	6.860
SFRB18	0.064	2.492	2.492	2.492	0.036	2.492
WWC15	0.290	13.640	13.644	13.644	0.094	13.562
	Power (%)					
CH06	6.420	96.688	94.484	94.484	1.462	91.098
MV04	70.418	87.260	93.032	93.032	34.934	96.542
N10	2.602	78.130	74.868	74.868	1.982	72.368
R08	0.846	66.926	36.450	36.450	6.672	13.196
SFRB18	1.780	93.862	93.580	93.580	0.458	89.786
WWC15	5.260	93.916	94.116	94.116	0.242	93.876

Table 2: Average empirical type I error rate (when $d = \gamma = 0.4$) and power (when $d = 0.4$ and $\gamma = 0.2$) of time series model for the six tests at bandwidth $\eta_m = 0.65, 0.75$ and correlation $\rho = 0.0, 0.4, 0.9$

Type I error (%)						
Method	$\rho = 0.0$		$\rho = 0.4$		$\rho = 0.9$	
	0.65	0.75	0.65	0.75	0.65	0.75
CH06	8.748	7.412	8.394	7.036	8.144	6.850
MV04	3.522	1.030	6.766	1.688	19.394	7.480
N10	7.954	7.512	8.234	7.678	8.662	7.908
R08	13.314	6.588	10.622	6.190	8.974	5.962
SFRB18	3.612	3.040	3.612	3.040	3.612	3.040
WWC15	15.992	10.384	15.996	10.386	15.922	10.500
	Power (%)					
CH06	34.196	45.074	25.640	34.012	17.812	32.640
MV04	8.584	2.930	11.182	5.708	15.640	15.172
N10	40.304	39.552	39.724	38.946	31.034	30.176
R08	1.852	6.148	2.030	2.848	5.758	4.914
SFRB18	22.684	34.390	22.626	34.222	20.508	31.648
WWC15	52.828	49.374	52.628	49.218	47.804	42.108

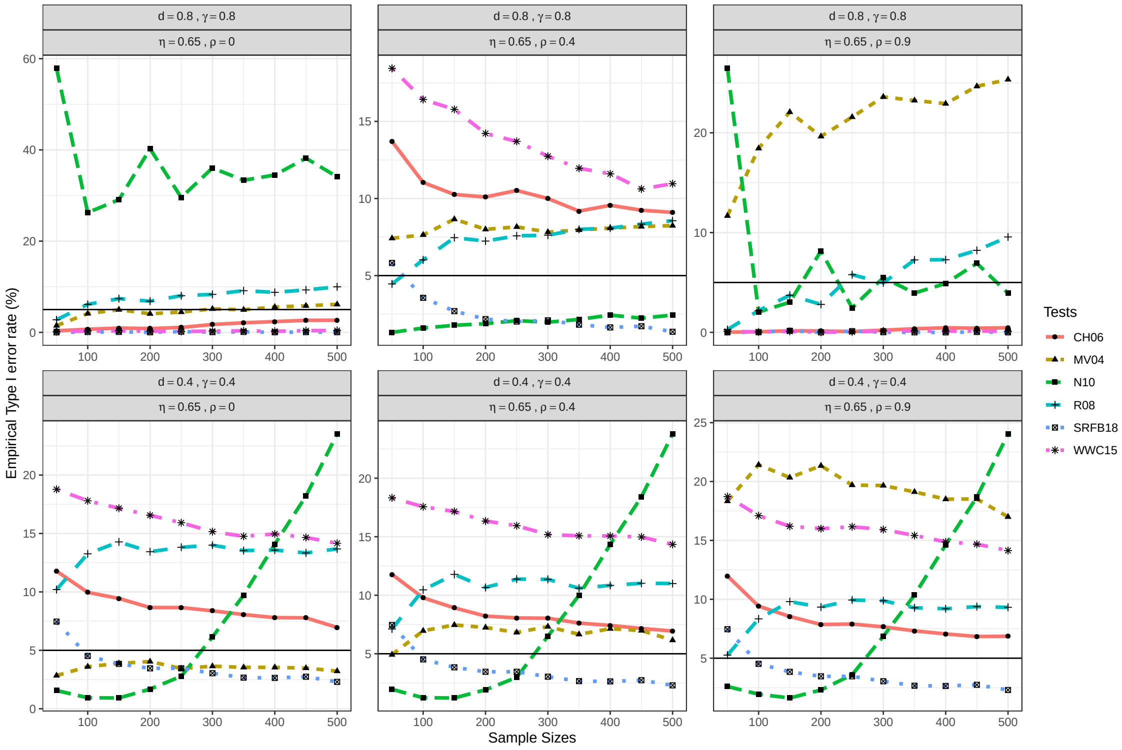

Figure 3: Empirical type I error rate (%) for time series model at bandwidth $\eta_m = 0.65$, $d = \gamma = 0.8$, $d = \gamma = 0.4$ and (i.) $\rho = 0$, (ii.) $\rho = 0.4$, (iii.) $\rho = 0.9$

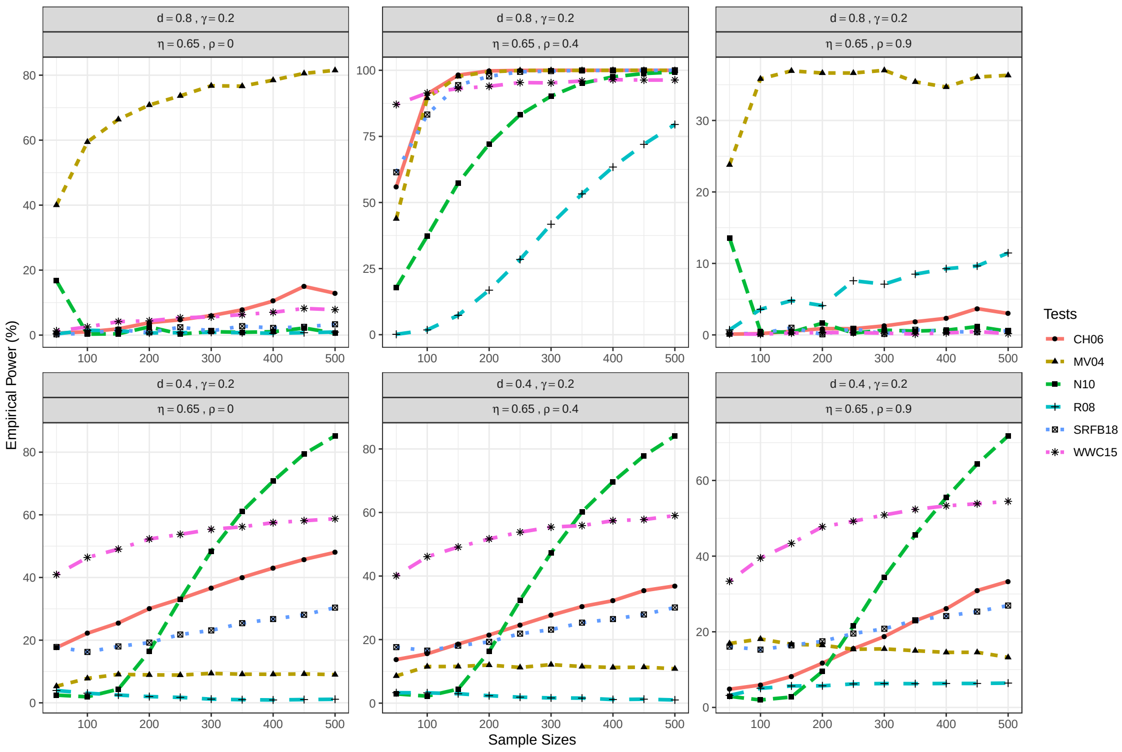

Figure 4: Empirical power (%) for time series model at bandwidth $\eta_m = 0.65$, $d = 0.8, \gamma = 0.2$, $d = 0.4, \gamma = 0.2$and (i.) $\rho = 0$, (ii.) $\rho = 0.4$, (iii.) $\rho = 0.9$

Furthermore, increasing the correlation of the residuals reduces the empirical type I errors for the methods of Chen and Hurvich (2006), Robinson (2008a) and Wang *et al.* (2015) while it increases the empirical type I errors for the methods of Marmol and Velasco (2004) and Nielsen (2010).

On the other hand, the results observed for $d = \gamma = 0.4$, which addresses cases with $d < 0.5$ showed similar results with $d > 0.5$ ($d = \gamma = 0.8$) except for the method of Marmol and Velasco (2004) which is now conservative and the method of Nielsen (2010) which is now restrictive as sample size increases.

Figure 2 presents the simulation results for the empirical power, which was used to assess the usability of the tests at the control level and bandwidth 0.75. A test is usable if its empirical power converges to 100% as sample size increases. At $d = 0.8$ and $\gamma = 0.2$, the results observed indicate that the methods of Chen and Hurvich (2006), Marmol and Velasco (2004), Nielsen (2010), Souza *et al.* (2018) and Wang *et al.* (2015) are usable at $T \geq 100$ and all correlation levels used. Again, as observed with empirical Type I error rate, the method of Robinson (2008a) was found to be unusable at smaller sample sizes and high correlation level. Similar results was observed when $d < 0.5$ ($d = 0.4$ and $\gamma = 0.2$) except that the method of Marmol and Velasco (2004) joined the group of unusable test at various correlation levels.

Figure 3 presents the simulation results for the empirical type I error rate at the control level and bandwidth 0.65. At $d = 0.8, \gamma = 0.8$, the results showed that only the methods of Marmol and Velasco (2004) appear to be valid at $\rho = 0$, while other methods underestimates the imposed threshold except the method of Robinson (2008a) that overestimates it as sample size increases. At $\rho = 0.4$, the methods of Nielsen (2010) and Souza *et al.* (2018) underestimate the threshold while other methods overestimates it. At $\rho = 0.9$, the performance of the tests appear to be better except for the method of Marmol and Velasco (2004). Figure 4 presents the simulation results for the empirical power at the control level and bandwidth 0.65. The results showed that at $\rho = 0$ only the methods of Marmol and Velasco (2004) yielded reasonable power while other methods are not usable. At $\rho = 0.4$, all the tests returned reasonable of as sample size increases. The performance of all the tests were found to be poor at $\rho = 0.9$.

For $d < 0.5$, similar results as observed when $d > 0.5$ were obtained except for the method of Nielsen (2010) whose empirical type I error rate increases with increase in sample size. In addition, the empirical type I error rates for all the tests were closer to the 5% threshold when $d < 0.5$. The preferable tests here include Chen and Hurvich (2006), Robinson (2008a) and Souza *et al.* (2018) which are more stable at various correlation levels when comapred to others.

Table 1 presents the overall results for the time series model when $d = \gamma = 0.8$ and $d = 0.8, \gamma = 0.2$. All the six tests achieved high power at bandwidth $\eta_m = 0.75$ across various correlation levels. In contrast for bandwidth $\eta_m = 0.65$, the six tests return low power except when correlation $\rho = 0.4$. Similarly, for the empirical Type I error rate, the performance at $\eta_m = 0.75$ is better for most of the tests compared to at $\eta_m = 0.65$. Thus, the final conclusion about the best test would be drawn using $\eta_m = 0.75$ results. Therefore, at $\eta_m = 0.75$, the uniformly most powerful test that maintains the 5% size, highest power and robustness to correlation levels is Souza *et al.* (2018).

Table 2 presents the overall results for the time series model when $d = \gamma = 0.4$ and $d = 0.4, \gamma = 0.2$. Most of the tests achieved their highest power at bandwidth $\eta_m = 0.75$ across various correlation levels. In contrast for bandwidth $\eta_m = 0.65$, the six tests return low power except for the methods of Wang *et al.* (2015), Nielsen (2010) and Marmol and Velasco (2004). Similarly, for the empirical Type I error rate, the performance at $\eta_m = 0.75$ is better for most of the tests compared to at $\eta_m = 0.65$. Thus, the final conclusion about the best test would also be drawn using $\eta_m = 0.75$ results. Therefore, at $\eta_m = 0.75$, the uniformly most powerful test that maintains the 5% size, average power and robustness to correlation levels is Souza *et al.* (2018). In terms of power, the most powerful test is Wang *et al.* (2015).

4.2. Scenario 2 (moderate): simulation procedure for homogenous panel

We assume the following balanced fixed effect panel model with n the number of cross-sectional units, t the time unit such that $T = n \times t$ is the total sample size.

$$\begin{aligned} y_{it} &= \mu + \beta x_{it} + \Delta^{-\gamma}\epsilon_{1it} \\ x_{it} &= \Delta^{-d}\epsilon_{2it} \end{aligned} \tag{19}$$

where μ is the constant intercept across the units $i = 1, 2, \ldots, n$, $\beta = -1$, $\epsilon_{it} = (\epsilon_{1it}, \epsilon_{2it})'$ being a Gaussian white noise with $E(\epsilon_{it}) = 0$, $Var(\epsilon_{1it}) = Var(\epsilon_{2it}) = 1$ and $Cov(\epsilon_{1it}, \epsilon_{2it}) = \rho$. We consider cases with $\rho = 0, 0.4, 0.9$, sample sizes $T = 50, 100, 150, 200, 250, 300, 350, 400, 450, 500$ and bandwidths power $\eta_m = 0.65, 0.75$ separately. We set $d = 0.4, 0.8$ and $\gamma = 0.8, 0.2$ for each d. The results for bandwidth power $\eta_m = 0.75$ were presented in Figure 3 - 4 while results for $\eta_m = 0.65$ are shown in Figure 5 - 6. For all tests, type I error was fixed at 5%.

Table 3: Average empirical type I error rate (when $d = \gamma = 0.8$) and power (when $d = 0.8$ and $\gamma = 0.2$) of homogenous panel model for the six tests at bandwidth $\eta_m = 0.65, 0.75$ and correlation $\rho = 0.0, 0.4, 0.9$

	Type I error rate (%)					
Method	$\rho = 0$		$\rho = 0.4$		$\rho = 0.9$	
	0.65	0.75	0.65	0.75	0.65	0.75
CH06	1.956	2.578	1.730	2.322	1.704	2.248
MV04	4.132	0.320	8.554	0.802	32.650	7.180
N10	84.220	88.606	83.678	88.332	83.068	87.944
R08	6.020	2.082	2.594	1.494	1.562	1.278
SFRB18	0.062	0.008	0.062	0.008	0.062	0.008
WWC15	0.030	0.002	0.030	0.002	0.030	0.002
	Power (%)					
CH06	99.478	99.584	99.370	99.486	99.264	99.442
MV04	99.072	94.382	98.596	92.308	97.940	89.524
N10	99.732	99.834	99.624	99.780	99.482	99.722
R08	39.082	80.636	28.370	73.620	9.626	37.586
SFRB18	71.128	85.524	70.610	85.328	67.524	83.696
WWC15	64.478	59.244	62.774	56.692	63.028	55.502

Figure 5 presents the simulation results for the empirical type I error rate at the moderate level (homogenous panel model) and bandwidth 0.75. At $d = \gamma = 0.8$, the results showed that all the tests underestimates the 5% threshold imposed at all levels of correlation except for the method of Marmol and Velasco (2004) at $\rho = 0.9$ and Nielsen (2010) at all correlation levels. At $d = \gamma = 0.4$, the results observed differs from when $d = \gamma = 0.8$ as the methods of Chen and Hurvich (2006) and Robinson (2008a) slightly overestimates the threshold imposed while other tests underestimates and Nielsen (2010) produces empirical type I error rates that increases with sample size. On the other hand, for the power when $d = 0.8, \gamma = 0.2$, Figure 6 shows that the group of tests with high power are Nielsen (2010), Marmol and Velasco (2004) and Chen and Hurvich (2006) while tests with moderate power are Robinson (2008a), Souza *et al.* (2018) and Wang *et al.* (2015). Similarly, when $d = 0.4, \gamma = 0.2$, only Chen and Hurvich (2006) and Nielsen (2010) returned high power while all other tests returned low power at various correlation levels.

Figure 7 presents the simulation results for the empirical type I error rate at the moderate level (homogenous panel model) and bandwidth 0.65. The results when $d = \gamma = 0.8$ showed that the methods of Chen and Hurvich (2006), Souza *et al.* (2018) and Wang *et al.* (2015) underestimates the 5% threshold imposed at all levels of correlation. The methods of Nielsen (2010), Robinson (2008a) and Marmol and Velasco (2004) overestimates the 5% threshold imposed at all levels of correlation. For $d = \gamma = 0.4$, the methods of Nielsen (2010); Chen and

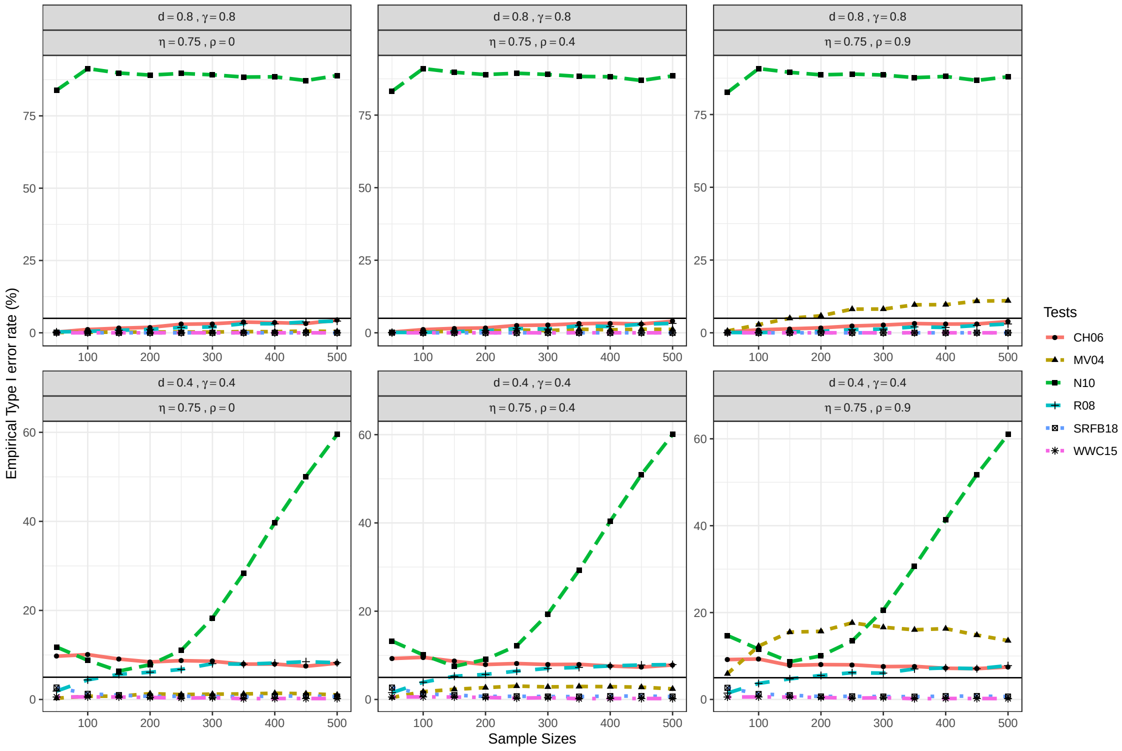

Figure 5: Empirical type I error rate (%) for homogenous panel model at bandwidth $\eta_m = 0.75$, $d = \gamma = 0.8$, $d = \gamma = 0.4$ and (i.) $\rho = 0$, (ii.) $\rho = 0.4$, (iii.) $\rho = 0.9$

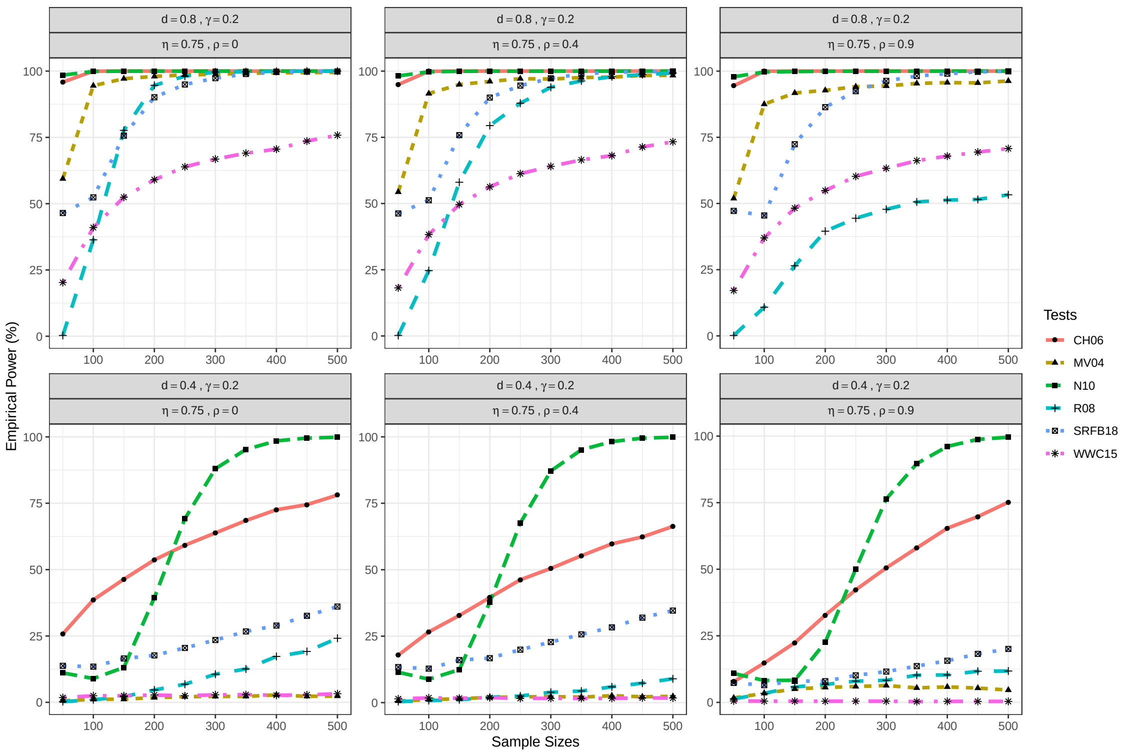

Figure 6: Empirical power (%) for homogenous panel model at bandwidth $\eta_m = 0.75$, $d = 0.8$,$\gamma = 0.2$, $d = 0.4$,$\gamma = 0.2$ and (i.) $\rho = 0$, (ii.) $\rho = 0.4$, (iii.) $\rho = 0.9$

Table 4: Average empirical type I error rate (when $d = \gamma = 0.4$) and power (when $d = 0.4$ and $\gamma = 0.2$) of homogenous panel model for the six tests at bandwidth $\eta_m = 0.65, 0.75$ and correlation $\rho = 0.0, 0.4, 0.9$

Type I error (%)						
Method	$\rho = 0$		$\rho = 0.4$		$\rho = 0.9$	
	0.65	0.75	0.65	0.75	0.65	0.75
CH06	9.474	8.612	9.162	8.214	9.004	7.926
MV04	5.610	1.034	11.122	2.408	31.282	14.434
N10	25.820	24.174	27.052	25.208	28.442	26.384
R08	11.166	6.568	8.754	6.042	7.216	5.678
SFRB18	1.290	0.982	1.290	0.982	1.290	0.982
WWC15	1.618	0.368	1.618	0.368	1.618	0.368
	Power (%)					
CH06	45.378	58.106	34.700	45.714	27.260	43.882
MV04	4.642	1.870	5.792	1.878	13.794	4.934
N10	63.276	62.304	62.634	61.786	57.182	56.064
R08	1.060	9.864	1.114	3.724	6.504	7.754
SFRB18	15.822	22.970	15.194	22.214	8.466	11.910
WWC15	7.350	2.646	5.248	1.658	2.572	0.432

Hurvich (2006); Marmol and Velasco (2004); Robinson (2008a)overestimates the true type I error rate while the methods of Wang *et al.* (2015); Souza *et al.* (2018) underestimates it. Figure 8 shows that for $d = 0.8, \gamma = 0.2$, the group of tests with high power are Nielsen (2010), Marmol and Velasco (2004) and Chen and Hurvich (2006) while tests with moderate power are Robinson (2008a), Souza *et al.* (2018) and Wang *et al.* (2015). On the other hand for $d = 0.4, \gamma = 0.2$, only Chen and Hurvich (2006) and Nielsen (2010) returned high power while all other tests returned low power at various correlation levels.

Table 3 present the overall results for the homogenous panel model when ($d = \gamma = 0.8$ and $d = 0.8, \gamma = 0.2$). All the six tests achieved high power at the two bandwidth levels $\eta_m = 0.65, 0.75$ across various correlation levels. Similarly, for the empirical type I error rate, the performance at $\eta_m = 0.75$ is better for most of the tests compared to at $\eta_m = 0.65$. Thus, the final conclusion about the best test would be drawn using $\eta_m = 0.75$ results. Therefore, at $\eta_m = 0.75$, the uniformly most powerful test that maintains the 5% size, highest power and robustness to correlation levels is Chen and Hurvich (2006).

Table 4 present the overall results for the homogenous panel model when ($d = \gamma = 0.4$ and $d = 0.4, \gamma = 0.2$). Similar results as in ($d = \gamma = 0.8$ and $d = 0.8, \gamma = 0.2$) were equally observed.

4.3. Scenario 3 (high): simulation procedure for heterogenous panel

We assume the following balanced fixed effect panel model with n the number of cross-sectional units, t the time unit such that $T = n \times t$ is the total sample size.

$$\begin{aligned} y_{it} &= \mu_i + \beta x_{it} + \Delta^{-\gamma}\epsilon_{1it} \\ x_{it} &= \Delta^{-d}\epsilon_{2it} \end{aligned} \tag{20}$$

where $\mu_i = i \; \forall i \in 1, \ldots, n$ implying $\boldsymbol{\mu} = [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]'$ is the vector of intercept across the units, $\beta = -1$, $\epsilon_{it} = (\epsilon_{1it}, \epsilon_{2it})'$ being a Gaussian white noise with $E(\epsilon_{it}) = 0$, $Var(\epsilon_{1it}) = Var(\epsilon_{2it}) = 1$ and $Cov(\epsilon_{1it}, \epsilon_{2it}) = \rho$. We consider cases with $\rho = 0, 0.4, 0.9$, sample sizes $T = 50, 100, 150, 200, 250, 300, 350, 400, 450, 500$ and bandwidths power $\eta_m = 0.65, 0.75$ separately. We set $d = 0.4, 0.8$ and $\gamma = 0.8, 0.2$ for each d. The results for bandwidth power $\eta_m = 0.75$ were presented in Figure 9 - 10 while results for $\eta_m = 0.65$ are shown in Figure 11 - 12. For all tests, type I error was fixed at 5%.

Table 5: Average empirical type I error rate (when $d = \gamma = 0.8$) and power (when $d = 0.8$ and $\gamma = 0.2$) of heterogenous panel model for the six tests at bandwidth $\eta_m = 0.65, 0.75$ and correlation $\rho = 0.0, 0.4, 0.9$

Type I error rate (%)						
Method	$\rho = 0$		$\rho = 0.4$		$\rho = 0.9$	
	0.65	0.75	0.65	0.75	0.65	0.75
CH06	1.536	2.178	1.144	1.764	0.238	0.418
MV04	4.674	0.524	8.514	0.974	21.318	3.498
N10	35.918	42.204	30.720	36.760	6.730	8.164
R08	7.670	2.614	3.796	1.966	5.204	3.476
SFRB18	0.064	0.018	0.070	0.018	0.036	0.004
WWC15	0.290	0.046	0.188	0.028	0.094	0.020
	Power (%)					
CH06	6.420	47.658	3.496	33.948	1.462	18.852
MV04	70.418	41.738	54.928	27.392	34.934	15.766
N10	2.602	1.686	2.298	1.299	1.982	0.988
R08	0.846	24.412	1.792	11.171	6.672	4.570
SFRB18	1.780	11.696	1.384	9.742	0.458	2.508
WWC15	5.260	12.370	1.854	4.980	0.242	1.182

Figure 9 presents the simulation results for the empirical type I error rate at the high level (heterogenous panel model) and bandwidth 0.75 when $d = \gamma = 0.8$. The results showed that all the tests underestimates the 5% threshold imposed at all levels of correlation except for the method of Nielsen (2010) at all correlation levels. For $d = \gamma = 0.4$, all the tests overestimates the imposed 5% threshold except the methods of Nielsen (2010); Souza *et al.* (2018). On the other hand, for the power, Figure 10 shows that when $d = 0.8, \gamma = 0.2$, the group of tests with high power are Marmol and Velasco (2004) and Chen and Hurvich (2006) while tests with moderate through low power are Robinson (2008a), Souza *et al.* (2018),Wang *et al.* (2015) and Nielsen (2010). In constrast, when $d = 0.4, \gamma = 0.2$, the group of tests with high power are Wang *et al.* (2015); Marmol and Velasco (2004); Robinson (2008a); Chen and Hurvich (2006) while Nielsen (2010); Souza *et al.* (2018) are again the lowest in terms of power.

Figure 11 presents the simulation results for the empirical type I error rate at the high level (heterogenous panel model) and bandwidth 0.65. The results at $d = \gamma = 0.8$ showed that the the methods of Chen and Hurvich (2006), Souza *et al.* (2018) and Wang *et al.* (2015) underestimates the 5% threshold imposed at all levels of correlation. The methods of Nielsen (2010), Robinson (2008a) and Marmol and Velasco (2004) overestimates the 5% threshold imposed at all levels of correlation. For $d = \gamma = 0.4$, all the tests overestimates the imposed 5% threshold except the methods of Nielsen (2010); Souza *et al.* (2018). Figure 12 shows that the only test with high power is Marmol and Velasco (2004) while other tests returned low power at various correlation levels. In constrast, when $d = 0.4, \gamma = 0.2$, the group of tests with high power are Wang *et al.* (2015); Marmol and Velasco (2004); Robinson (2008a); Chen and Hurvich (2006) while Nielsen (2010); Souza *et al.* (2018) are again the lowest in terms of power.

Table 5 presents the overall results for the heterogenous panel model when $d = 0.8, \gamma = 0.2$.

Table 6: Average empirical type I error rate (when $d = \gamma = 0.4$) and power (when $d = 0.4$ and $\gamma = 0.2$) of heterogenous panel model for the six tests at bandwidth $\eta_m = 0.65, 0.75$ and correlation $\rho = 0.0, 0.4, 0.9$

Type I error (%)						
Method	$\rho = 0$		$\rho = 0.4$		$\rho = 0.9$	
	0.65	0.75	0.65	0.75	0.65	0.75
CH06	63.700	68.068	43.042	46.326	19.486	21.014
MV04	95.334	97.548	94.802	97.318	94.062	96.576
N10	0.064	0.034	0.078	0.036	0.108	0.028
R08	86.306	82.232	86.724	85.954	93.236	94.856
SFRB18	0.014	0.012	0.008	0.004	0.000	0.000
WWC15	97.126	97.466	94.530	94.858	87.170	87.424
	Power (%)					
CH06	63.136	61.892	40.880	34.454	16.494	9.396
MV04	98.392	99.018	97.050	98.722	92.714	97.470
N10	0.046	0.010	0.044	0.012	0.058	0.018
R08	88.372	82.174	90.274	88.074	94.796	95.894
SFRB18	0.012	0.014	0.010	0.014	0.000	0.000
WWC15	97.434	97.548	94.748	94.506	86.598	83.018

All the six tests achieved low power at the two bandwidth levels $\eta_m = 0.65, 0.75$ across various correlation levels. Similarly, for the empirical type I error rate, the performance at $\eta_m = 0.75$ is better for most of the tests compared to at $\eta_m = 0.65$. Thus, the final conclusion about the best test would be drawn using $\eta_m = 0.75$ results. Therefore, at $\eta_m = 0.75$, the uniformly most powerful test that maintains the 5% size, highest power and moderate robustness to correlation levels is Chen and Hurvich (2006).

Table 6 presents the overall results for the heterogenous panel model when $d = 0.4, \gamma = 0.2$. All the six tests except Nielsen (2010); Souza *et al.* (2018) achieved high power at the two bandwidth levels $\eta_m = 0.65, 0.75$ across various correlation levels. Similarly, for the empirical type I error rate, the performance at $\eta_m = 0.75$ is not different from $\eta_m = 0.65$ with all tests overestimating the true type I error rates across various correlation levels. Therefore, none of the six tests is valid for testing fractional cointeration in heterogenous panel settings when $d < 0.5$. However, all the tests except Nielsen (2010); Souza *et al.* (2018) are usable for testing fractional cointegration in heterogenous panel settings when $d < 0.5$.

5. Application to real-life dataset

The datasets used here were drawn from Yahoo Finance and Kenneth French's Data Library. Five Industries portfolios (Cnsmr, Manuf, HiTec, Hlth, and Other) for 240 months were extracted from Kenneth French's Data Library. This dataset was used to compute the industry realized volatility. The market volatility data was extracted from Yahoo Finance for three composite portfolios (NYSE, NASDAQ and AMEX). The market portfolios were aggregated to be used as constant input for the realized industry portfolios. We let IV_{it} $i = 1, 2, 3, 4, 5$, $t = 1, \ldots, 240$ represent industry volatility and MV_{it} represent market volatility. The associated fractional cointegrated panel model is given by

$$\begin{aligned} IV_{it} &= \mu_i + \beta MV_{it} + \Delta^{-\gamma}\epsilon_{1it} \\ MV_{it} &= \Delta^{-d}\epsilon_{2it}. \end{aligned} \tag{21}$$

We estimated the fractional cointegration parameters d and γ using bandwidth $\eta_m = 0.75$

for each industry and for the pooled industries (Panel). It is essential to test the equality of d across portfolios to ensure the validity of pooling. The tests of Robinson and Yajima (2002) and Nielsen and Shimotsu (2007) were applied and the estimated result observed is ($T_{stat} = 0.378, p - value = 0.3528$). The result shows that the null hypothesis of equality of d across various portfolios holds. Furthermore, the test for null hypothesis of no fractional cointegration was conducted using the six tests and the results reported in Table 7.

Table 7: Estimate of d, γ, β and test of hypothesis of no fractional cointegration for the five industries and market average

Method/Parameter	Market	Cnsmr	Manuf	HiTec	Hlth	Other	Panel
$\hat{d}$	0.55	0.55	0.52	0.61	0.46	0.74	0.58
$\hat{\gamma}$		0.20	0.42	0.87	0.32	0.34	0.56
$\hat{\beta}$		0.746	0.980	1.110	0.680	1.261	0.955
$SE(\hat{\beta})$		(0.016)	(0.022)	(0.043)	(0.028)	(0.029)	(0.016)
$rank : (\hat{r})$		1.000	1.000	1.000	1.000	1.000	1.000
CH06	Statistic	2.592	0.677	-0.566	1.186	3.621	-0.222
	Decision	TRUE	FALSE	FALSE	FALSE	TRUE	FALSE
MV04	Statistic	0.027	0.129	2.702	0.753	171.286	13.429
	Decision	FALSE	FALSE	FALSE	FALSE	TRUE	TRUE
N10	Statistic	3.677	3.339	2.970	3.302	3.604	3.756
	Decision	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE
R08	Statistic	2.871	0.362	5.000	0.385	1.925	4.073
	Decision	FALSE	FALSE	TRUE	FALSE	FALSE	TRUE
SFRB18	Statistic	3.076	0.205	-1.136	0.449	3.784	3.268
	Decision	TRUE	FALSE	FALSE	FALSE	TRUE	TRUE
WWC15	Statistic	7.540	2.162	0.786	2.797	12.691	1.279
	Decision	TRUE	TRUE	FALSE	TRUE	TRUE	FALSE

By default, realized industry volatility is cointegrated with market volatility if $\hat{\gamma} < \hat{d}$. Table 4 shows that $\hat{\gamma} < \hat{d}$ for most of the industries except HiTec industry with $\hat{\gamma} > \hat{d}$. Thus, expectedly, the null hypothesis of no fractional cointegration should be rejected for industries with $\hat{\gamma} < \hat{d}$ if the difference is significant. The methods of Chen and Hurvich (2006) and Souza *et al.* (2018) behaves similarly with rejection of null hypothesis when there is large difference between the memory parameters for the industries. The presence of fractional cointegration in two industries could not trigger panel fractional cointegration with Chen and Hurvich (2006) while it triggers with Souza *et al.* (2018). The results of Marmol and Velasco (2004) and Wang *et al.* (2015) are not reliable as high false rejections were observed in the various simulation scenarios. In addition, rejection of no fractional cointegration in three of the five industries with Wang *et al.* (2015) could not lead to overall rejection of the null hypothesis of no fractional panel cointegration. The results from methods of Robinson (2008a) and Nielsen (2010) are also not reliable as they are not applicable to nonstationary series with $\hat{d} > 0.5$. Therefore, further exploration of the methods of Chen and Hurvich (2006) and Souza *et al.* (2018) is necessary to resolve if truly a fractional panel cointegration exists in the dataset.

Note: A decision is TRUE in Table 4 if the null hypothesis of no fractional cointegration is rejected and FALSE if otherwise.

6. Conclusion

The objective of this paper is to compare the applicability of six existing semiparametric fractional cointegration tests in time series and panel data settings. Leschinski *et al.* (2020) already presented an extensive comparative simulation study of the tests in time series data and observed that the best bandwidth is 0.75 and the best test at both $d < 0.5$ and $d > 0.5$ is Souza *et al.* (2018). The tests were adjudged based on their ability to achieve the imposed type I error, high power and robustness to different correlation structures. Based on the results of the simulation conducted in this paper, it can be concluded that the best test for examining fractional cointegration in time series data is Souza *et al.* (2018). This findings corroborates the results observed in Leschinski *et al.* (2020). On the other hand, the simulation results also revealed that the best test for examining panel fractional cointegration is Chen and Hurvich (2006). Furthermore, the real-life industries and market volatility data analysis corroborates the applicability of the two methods. Also, most of the tests performed better with bandwidth $\eta_m = 0.75$ compared to $\eta_m = 0.65$.

Overall, the various results observed with fractional cointegration in panel settings showed that there is still room for improvement as even the best test Chen and Hurvich (2006) could not achieve 60% power under the heterogenous panel model condition. This, therefore, implies that further modification of the methods of Chen and Hurvich (2006) and Souza *et al.* (2018) is vital to achieving improved applicability in fixed and random effects panel settings.

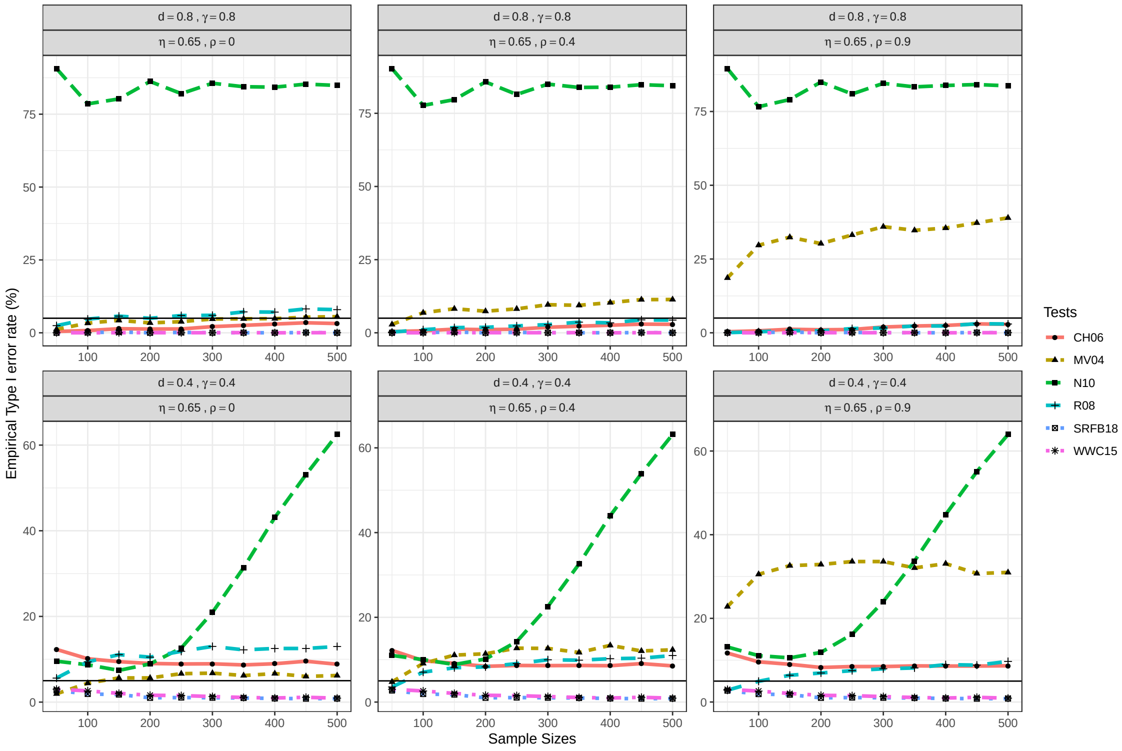

Figure 7: Empirical type I error rate (%) for homogenous panel model at bandwidth $\eta_m = 0.65$, $d = \gamma = 0.8$, $d = \gamma = 0.4$ and (i.) $\rho = 0$, (ii.) $\rho = 0.4$, (iii.) $\rho = 0.9$

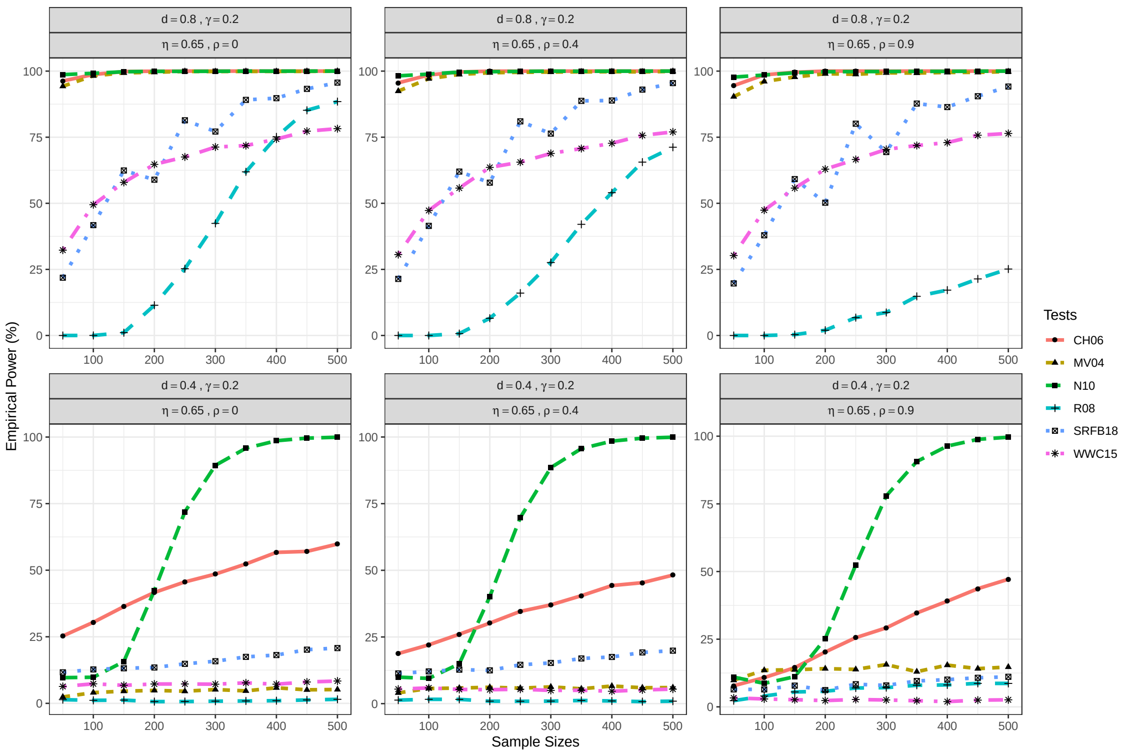

Figure 8: Empirical power (%) for homogenous panel model at bandwidth $\eta_m = 0.65$, $d = 0.8$,$\gamma = 0.2$, $d = 0.4$,$\gamma = 0.2$ and (i.) $\rho = 0$, (ii.) $\rho = 0.4$, (iii.) $\rho = 0.9$

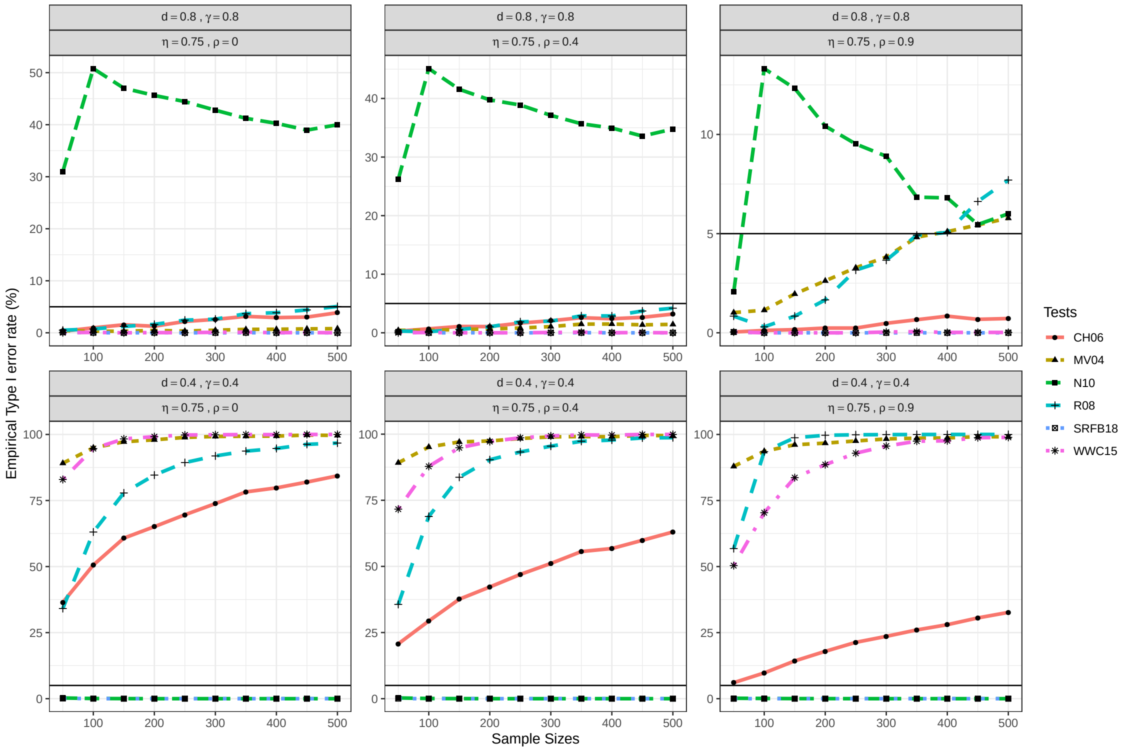

Figure 9: Empirical type I error rate (%) for heterogenous panel model at bandwidth $\eta_m = 0.75$, $d = \gamma = 0.8$, $d = \gamma = 0.4$ and (i.) $\rho = 0$, (ii.) $\rho = 0.4$, (iii.) $\rho = 0.9$

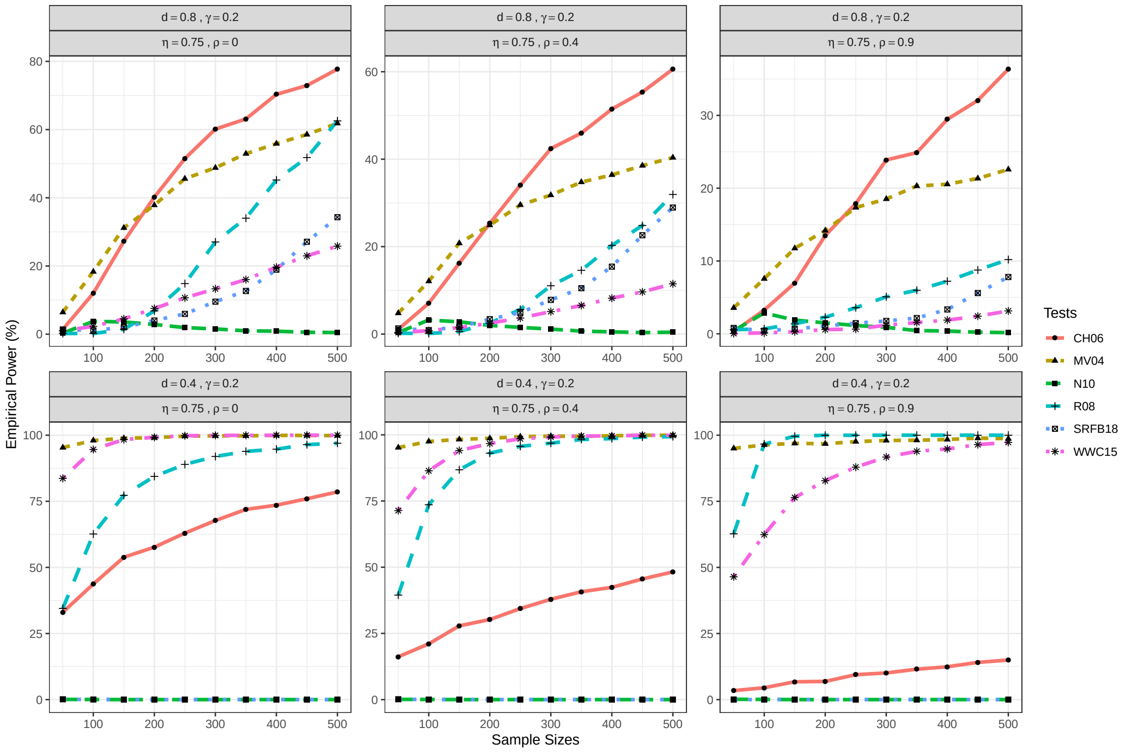

Figure 10: Empirical power (%) for heterogenous panel model at bandwidth $\eta_m = 0.75$, $d = 0.8$,$\gamma = 0.2$, $d = 0.4$,$\gamma = 0.2$ and (i.) $\rho = 0$, (ii.) $\rho = 0.4$, (iii.) $\rho = 0.9$

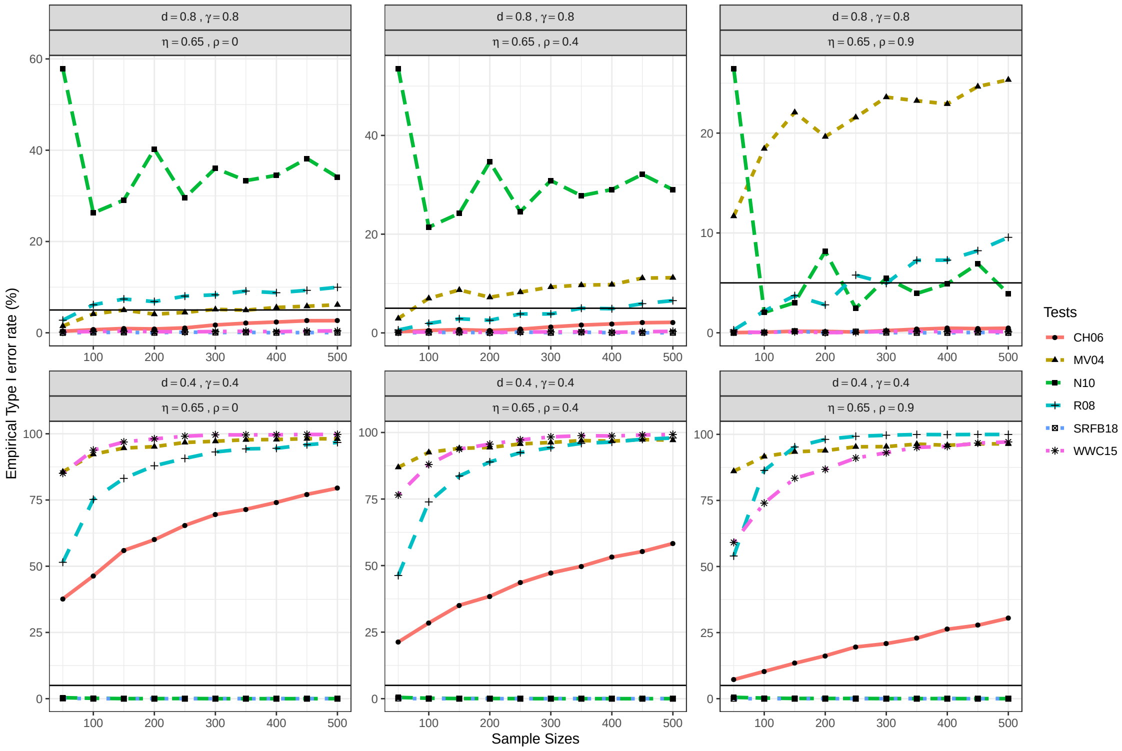

Figure 11: Empirical type I error rate (%) for heterogenous panel model at bandwidth $\eta_m = 0.65$, $d = \gamma = 0.8$, $d = \gamma = 0.4$ and (i.) $\rho = 0$, (ii.) $\rho = 0.4$, (iii.) $\rho = 0.9$

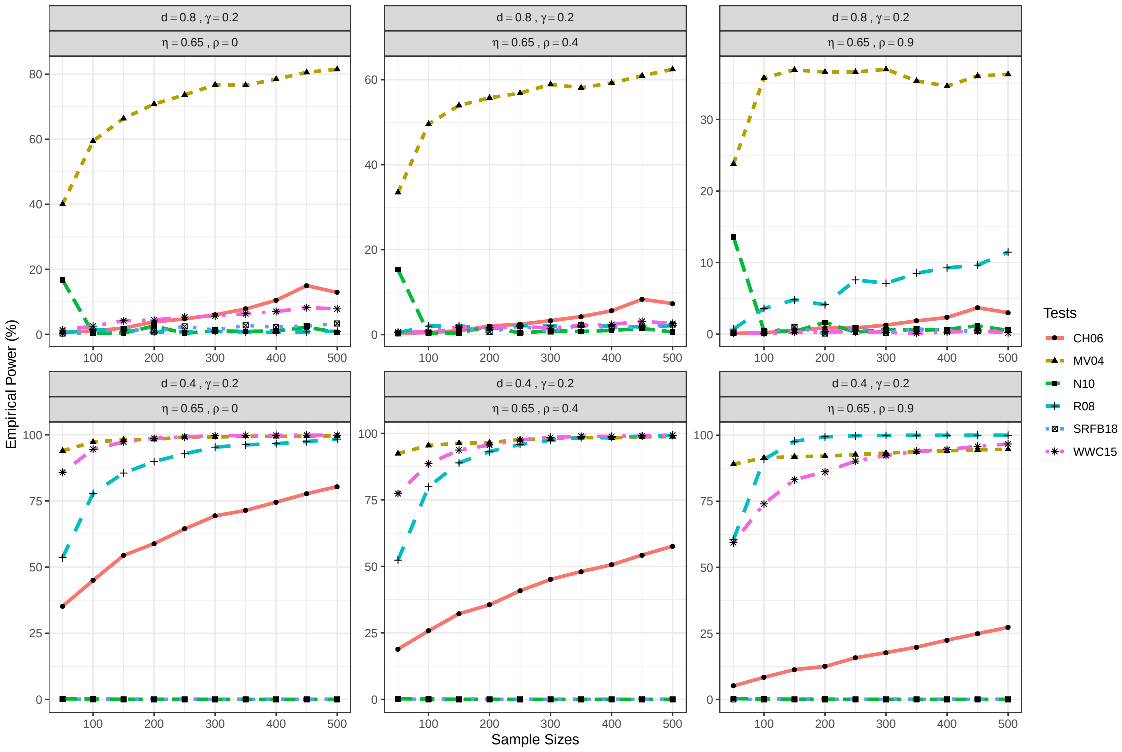

Figure 12: Empirical power (%) for heterogenous panel model at bandwidth $\eta_m = 0.65$, $d = 0.8$,$\gamma = 0.2$, $d = 0.4$,$\gamma = 0.2$ and (i.) $\rho = 0$, (ii.) $\rho = 0.4$, (iii.) $\rho = 0.9$

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Austrian Journal of Statistics http://www.ajs.or.at/
published by the Austrian Society of Statistics http://www.osg.or.at/
Volume 51 *Submitted:* 2020-05-20
July 2022 *Accepted:* 2021-02-26