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Bayes Prediction on Optimum SS-PALT in Generalized Inverted Exponential Distribution: A Two-Sample Approach

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Abstract

The generalized Inverted Exponential distribution is considered for the study on Optimum Step Stress Partially Accelerated Life Test (SS-PALT) based on different censoring patterns. The first-failure progressive censoring (FFPC) scheme and their special cases are used in the present study. A two-sample Bayes Prediction Bound Length (TS-BPBL) under SS-PALT on FFPC have been obtained and studied their properties by using different special cases of FFPC. Based on simulated and real data set, the properties of the ML estimates and the approximate confidence length under the normal approximation, also have been studied.

Keywords: approximate confidence length (ACL), first-failure progressive censoring (FFPC) scheme, generalized inverted exponential distribution, step stress partially accelerated life test (SS-PALT), two-sample Bayes prediction bound length (TS-BPBL)..

1. Introduction

The Exponential distribution is one of the widely used models in the life-testing experiment because of its simple mathematical practices and interesting properties. An inverted Exponential distribution was discussed by Lin, Duran, and Lewis (1989) by making a valuable changes in the Exponential distributionand and studied the ML estimation, confidence limits, UMVUE and the reliability function based on complete sample case. The generalization of the Exponential distribution was first discussed by Gupta and Kundu (1999) by introducing a shape parameter and named as the generalized Exponential distribution. Dey (2007) studied the inverted Exponential distribution under the Bayesian viewpoint by using different loss functions.

A little work has been done on the generalized Inverted Exponential distribution. Abouammoh and Alshingiti (2009) introduced the generalized inverted Exponential distribution by appending a shape parameter to the inverted Exponential distribution. They observed that, this distribution originated from the exponentiated Frechet distribution (Nadarajah and Kotz

(2003)). Due to its convenient structure of the distribution function, the generalized inverted Exponential distribution may be used in horse racing, queue theory, modeling wind speeds and much more. Prakash (2009) discussed some estimation of the parameters based on lower record values of the inverted exponential distribution.

Based on the hybrid censoring Dey and Pradhan (2014) have derived the maximum likelihood and Bayes estimates of the unknown parameters. Singh, Singh, and Kumar (2014) was discussed some Bayes estimation by using two different loss functions for generalized Inverted Exponential distribution. Various methods of estimation for unknown parameters of the concern distribution from a frequentist as well as Bayesian perspective was discussed by Ahmed (2017) recently.

The probability density and the cumulative distribution function of the generalized inverted Exponential distribution are given as

$$f\left(x;\sigma,\theta\right)=\frac{\sigma\theta}{x^2}\,exp\left(-\frac{\theta}{x}\right)\left(1-exp\left(-\frac{\theta}{x}\right)\right)^{\sigma-1}\;;\;\sigma>0,\theta>0,x>0 \tag{1}$$

and

$$F\left(x;\sigma,\theta\right)=1-\left(1-exp\left(-\frac{\theta}{x}\right)\right)^{\sigma}\;;\;\sigma>0,\theta>0,x>0. \tag{2}$$

Here, the parameter σ and θ are known as the shape and scale parameter respectively. The reliability function and the Hazard function of the distribution given in Eq. (1) are obtained as

$$\Psi(x)=\left(1-exp\left(-\frac{\theta}{x}\right)\right)^{\sigma}\;;\;\sigma>0,\theta>0,x>0 \tag{3}$$

and

$$\rho(x)=\frac{\sigma\theta}{x^2}\left(exp\left(\frac{\theta}{x}\right)-1\right)^{-1}\;;\;\sigma>0,\theta>0,x>0. \tag{4}$$

The main aim of the present study, is to get the two-sample Bayes Prediction Bound Length (TS-BPBL) for the generalized Inverted Exponential distribution. The optimum Step-Stress Partially Accelerated Life Test (SS-PALT) situation has been used under different special cases of FFPC. A comparison between TS-BPBL with an approximate confidence length, which is obtained by the method of maximum likelihood estimation with normal approximation have also discussed. The numerical illustration has presented on both the simulated and real data set.

2. SS-PALT under FFPC

In life testing product experiments, it is much difficult to gather lifetimes of extremely reliable products, having a very long lifespan. Because, under the normal operating conditions, a very few or even no failures may occur within a limited testing time interval. The accelerated life test or the partially accelerated life test criterion is very useful test criterion in such cases and, are provided significant reduction in the time and cost of the experiment.

In accelerated life test (ALT) criterion, all the test units are kept under a higher stress levels, but in partially accelerated life test (PALT) criterion, only a few test units from all the test units are kept under severe stress condition. See Abdel-Hamid and AL-Hussaini (2008) and Abdel-Hamid (2009) for the details regarding the ALT and PALT criterion. There are two different methods of stress loading in PALT, named as constant-stress and step-stress. The step-stress PALT is considered in the present study, and it permits the test, to be changed from the normal stress condition to the accelerated stress condition at a pre-assumed time.

Srivastava and Mittal (2010) have obtained the optimum step-stress partially accelerated life tests for the truncated logistic distribution under the censored data. Tangi, Guani, Xu, and Xu (2012) presents an optimum design for step-stress accelerated life tests based on Type-I censored data followed the two-parameter Weibull distribution. Recently, Prakash (2017 a) discussed about some bound lengths based on One-Sample Plan for Burr Type-XII distribution under the step-stress partially accelerated life tests.

Kamal, Zarrin, and Islam (2013) discussed some properties for the inverted Weibull distribution with application of constant-stress partially accelerated life test with Type-I censored data. Hyun and Lee (2015) presents some discussion on Bayes estimation followed the constant-stress partially accelerated life test for the log-Logistic distribution with censored data.

In SS-PALT, all n (say) test units are first run at the normal stress condition up to stress change time (say ϵ), and if it does not fail, then the test is changed to the accelerated stress condition and retained the test until all the units fail. If β is considered as the acceleration factor, then the tampered random variable model under SS-PALT for the lifetime of a unit (Y say) is defined as

$$X = \begin{cases} Y & ; \quad 0 < Y \leq \epsilon \\ \epsilon + \frac{Y-\epsilon}{\beta} & ; \quad Y > \epsilon; \end{cases} . \tag{5}$$

In the first-failure life test, researcher splits the test units into a number of groups and each groups are assembly of test units. Run all the units simultaneously by first-failure occurred in each group. Wu and Kus (2009) joined this with the Progressive censoring and named as first-failure progressive censoring (FFPC). The FFP censoring scheme has advantages in term of reducing test time, in which more items are used but only a few items are failed. Let us assume $(n \times k)$ live test units, in which k independent groups having n units within each groups are putting on a life test. Suppose that, $X_1^R < X_2^R < ... < X_m^R$ are the progressively first - failure censored order statistic of size m, with $R = (R_1, R_2, ..., R_m)$ pre assumed progressive censoring scheme. Following) (2017 (b), the joint probability density function under FFP censoring is defined as

$$L \propto \prod_{i=1}^{m} f\left(x_i^R; \sigma, \theta\right)\left(1 - F\left(x_i^R; \sigma, \theta\right)\right)^{k(R_i+1)-1}. \tag{6}$$

For $k = 1$, the Eq. (6) reflects the joint probability density function under the Progressive Type - II censoring. The FFP censoring scheme is combined here with SS - PALT, therefore, the Eq. (6) is re - written as

$$L \propto \prod_{i=1}^{m_1} f_1(x)\left(1 - F_1(x)\right)^{k(R_i+1)-1} \times \prod_{i=m_1+1}^{m} f_2(x)\left(1 - F_2(x)\right)^{k(R_i+1)-1} \tag{7}$$

Where,

$$f\left(x_i^R; \sigma, \theta\right) = \begin{cases} f_1(x) = \frac{\sigma\theta}{x^2} exp\left(-\frac{\theta}{x}\right)\left(1 - exp\left(-\frac{\theta}{x}\right)\right)^{\sigma-1} \\ f_2(x) = \frac{\sigma\theta\beta}{\tilde{X}^2} exp\left(-\frac{\theta}{\tilde{X}}\right)\left(1 - exp\left(-\frac{\theta}{\tilde{X}}\right)\right)^{\sigma-1} & ; \quad \tilde{X} = \epsilon + \beta(x - \epsilon) \end{cases} \tag{8}$$

Using Eq. (8) in Eq. (7), the required joint density function based on FFPC on SS - PALT is given as

$$L \propto \prod_{i=1}^{m_1} \left\{ \frac{\sigma\theta}{x_i^2} exp\left(-\frac{\theta}{x_i}\right)\left(1 - exp\left(-\frac{\theta}{x_i}\right)\right)^{\sigma k(R_i+1)-1} \right\}$$

$$\times \prod_{i=m_1+1}^{m}\left\{\frac{\sigma\theta\beta}{\tilde{X}_i^2}\, exp\left(-\frac{\theta}{\tilde{X}_i}\right)\left(1-exp\left(-\frac{\theta}{\tilde{X}_i}\right)\right)^{\sigma k(R_i+1)-1}\right\}$$

$$\Rightarrow L \propto \sigma^m\, \theta^m\, \beta^{m-m_1}\, T_0\left(\underline{x},\beta\right)\, e^{-\theta(T_1(\underline{x})+T_2(\underline{x},\beta))} \times e^{T_3(\underline{x})+T_4(\underline{x},\beta)} \tag{9}$$

where $T_0\left(\underline{x},\beta\right) = \prod_{i=m_1+1}^{m}\left(\tilde{X}_i^{-2}\right)$, $T_1\left(\underline{x}\right) = \sum_{i=1}^{m_1}\left(x_i^{-1}\right)$, $T_2\left(\underline{x},\beta\right) = \sum_{i=m_1+1}^{m}\left(\tilde{X}_i^{-1}\right)$, $T_3\left(\underline{x}\right) = \sum_{i=1}^{m_1}\left(\sigma k\left(R_i+1\right)-1\right) log\left(1-exp\left(-\frac{\theta}{x_i}\right)\right)$, $T_4\left(\underline{x},\beta\right) = \sum_{i=m_1+1}^{m}\left(\sigma k\left(R_i+1\right)-1\right)$ $log\left(1-exp\left(-\frac{\theta}{\tilde{X}_i}\right)\right)$ and $\tilde{X}_i = \epsilon+\beta\left(x_i-\epsilon\right)$.

3. Maximum likelihood interval estimation

We derive the maximum likelihood estimates using the expectation maximization algorithm and compute the observed information of the parameters that can be used for constructing asymptotic confidence intervals in the present section. The logarithm of Eq. (9) is obtained as

$$logL = m\, log\,\sigma + m\, log\,\theta + \left(m-m_1\right)\, log\,\beta + \log\, T_0\left(\underline{x},\beta\right)$$

$$-\theta\left(T_1\left(\underline{x}\right)+T_2\left(\underline{x},\beta\right)\right) + T_3\left(\underline{x}\right)+T_4\left(\underline{x},\beta\right). \tag{10}$$

Differentiating Eq. (10) with respect to the parameter θ and the acceleration factor β respectively, we get

$$\frac{\partial}{\partial\theta}logL = \frac{m}{\theta} - T_1\left(\underline{x}\right) - T_2\left(\underline{x},\beta\right) + \sum_{i=1}^{m_1}\left\{\frac{\left(\sigma k\left(R_i+1\right)-1\right)\, exp\left(-\frac{\theta}{x_i}\right)}{x_i\left(1-exp\left(-\frac{\theta}{x_i}\right)\right)}\right\}$$

$$+\sum_{i=m_1+1}^{m}\left\{\frac{\left(\sigma k\left(R_i+1\right)-1\right)\, exp\left(-\frac{\theta}{\tilde{X}_i}\right)}{\tilde{X}_i\left(1-exp\left(-\frac{\theta}{\tilde{X}_i}\right)\right)}\right\} \tag{11}$$

and

$$\frac{\partial}{\partial\beta}log\, L = \frac{m-m_1}{\beta} + \sum_{i=m_1+1}^{m}\left\{\frac{x_i-\epsilon}{\tilde{X}_i}\left(\frac{\theta}{\tilde{X}_i}\left(1+\frac{\left(\sigma k\left(R_i+1\right)-1\right)}{1-exp\left(\frac{\theta}{\tilde{X}_i}\right)}\right)-2\right)\right\}. \tag{12}$$

The second derivatives corresponding to the parameters θ and β are obtained respectively as

$$\frac{\partial^2}{\partial\theta^2}log\, L = -\frac{m}{\theta^2} - \sum_{i=1}^{m_1}\left\{\frac{\left(\sigma k\left(R_i+1\right)-1\right)\, exp\left(-\frac{\theta}{x_i}\right)}{x_i^2\left(1-exp\left(-\frac{\theta}{x_i}\right)\right)^2}\right\}$$

$$-\sum_{i=m_1+1}^{m}\left\{\frac{\left(\sigma k\left(R_i+1\right)-1\right)\, exp\left(-\frac{\theta}{\tilde{X}_i}\right)}{\tilde{X}_i^2\left(1-exp\left(-\frac{\theta}{\tilde{X}_i}\right)\right)^2}\right\},$$

$$\frac{\partial^2}{\partial\theta\partial\beta}logL = \sum_{i=m_1+1}^{m}\frac{x_i-\epsilon}{\tilde{X}_i^2}\left\{1-\sum_{i=m_1+1}^{m}\left(\sigma k\left(R_i+1\right)-1\right)\frac{\left(\tilde{X}_i-\theta\right)exp\left(\frac{\theta}{\tilde{X}_i}\right)-\tilde{X}_i}{\tilde{X}_i\left(exp\left(\frac{\theta}{\tilde{X}_i}\right)-1\right)^2}\right\}$$

and

$$\frac{\partial^2}{\partial \beta^2} logL = -\frac{m - m_1}{\beta^2} + 2 \sum_{i=m_1+1}^{m} \left(\frac{x_i - \epsilon}{\tilde{X}_i} \right)^2 \left(1 - \frac{\theta}{\tilde{X}_i} \right) + \sum_{i=m_1+1}^{m} \frac{\theta \left(x_i - \epsilon \right)^2}{\tilde{X}_i^3}$$

$$\times \left(\frac{\sigma k \left(R_i + 1 \right) - 1}{1 - exp\left(\frac{\theta}{\tilde{X}_i} \right)} \right) \left\{ \frac{\theta}{1 - exp\left(\frac{\theta}{\tilde{X}_i} \right)} + \frac{\theta}{\tilde{X}_i} - 2 \right\}.$$

Asymptotic co-variances and variances of the parameters under ML estimation are obtained from the inverse of the Fisher information matrix. The exact mathematical expressions are not possible to get, however, a numerical method have applied here for the numerical findings. The inverse of the Fisher information matrix is defined as

$$I = \begin{bmatrix} -\frac{\partial^2}{\partial \theta^2} logL & -\frac{\partial^2}{\partial \theta \partial \beta} logL \\ & \\ -\frac{\partial^2}{\partial \beta \partial \theta} logL & -\frac{\partial^2}{\partial \beta^2} logL \end{bmatrix}^{-1}_{\left(\hat{\theta}_{ML}, \hat{\beta}_{ML} \right)} \tag{13}$$

The second order derivatives involved the unknown parameters, so the Fisher information matrix can be obtained by replacing its ML estimators. A $(1 - \tau)\, 100\%$ improved approximate confidence limits for the parameters θ and β are obtained by following Meeker and Escobar (1998), and given respectively as

$$\left\{ \hat{\theta}_{ML}\, exp \left(\mp \frac{Z_{\tau/2} \sqrt{VAR \left(\hat{\theta}_{ML} \right)}}{\hat{\theta}_{ML}} \right) \right\} \tag{14}$$

and

$$\left\{ \hat{\beta}_{ML}\, exp \left(\mp \frac{Z_{\tau/2} \sqrt{VAR \left(\hat{\beta}_{ML} \right)}}{\hat{\beta}_{ML}} \right) \right\}. \tag{15}$$

Here, $Z_{\tau/2}$ is the percentile of the standard normal distribution with right - tail probability $\tau/2$. Here, the variances of the maximum likelihood estimators $\hat{\theta}_{ML}$ and $\hat{\beta}_{ML}$ are the principal diagonal elements of the matrix I respectively. The ML estimators and approximate confidence limits cannot be solved analytically; a numerical method applied here to solve these equations.

4. Two-sample Bayes prediction bound length

In this section we consider Bayesian inference to get the Bayes Prediction Bound Length under Two-Sample approach (TS-BPBL). The two-sample approach predict the j^{th} order statistic from future sample, based on an informative sample (See Prakash (2015) for more details).

It may be noted that, if both parameters σ and θ are unknown, no joint conjugate prior exists. In such a situation, there are some ways to choose the piece-wise independent priors. In present discussion, gamma prior have been selected for these parameters and given as

$$\pi_\theta \propto \theta^{\alpha-1}\, e^{-\theta}\, ;\ \alpha > 0 \tag{16}$$

and

$$\pi_\sigma \propto \sigma^{\gamma-1}\, e^{-\sigma}\, ;\ \gamma > 0. \tag{17}$$

A vague prior is assumed here for the acceleration factor β, so that the prior do not play any significant roles in the analyses. The vague prior for the parameter β and the joint prior distribution both are given respectively as

$$\pi_\beta = \beta^{-1}\ ;\ \beta > 0 \tag{18}$$

and

$$\pi_{(\theta,\sigma,\beta)} \propto \frac{\theta^{\alpha-1}\sigma^{\gamma-1}}{\beta}\, e^{-\theta-\sigma}. \tag{19}$$

Thus, the marginal posterior densities corresponding to the parameters θ, σ and β are obtained and given respectively as

$$\pi^*_\theta = \Omega \frac{\theta^{m+\alpha-1}}{e^{\theta(T_1(\underline{x})+1)}} \int_\sigma \frac{\sigma^{m+\gamma-1}}{e^{\sigma-T_3(\underline{x})}} \int_\beta \beta^{m-m_1-1} T_0\left(\underline{x},\beta\right)\ e^{T_4(\underline{x},\beta)-\theta T_2(\underline{x},\beta)}\, d\beta\, d\sigma \tag{20}$$

$$\pi^*_\beta = \Omega\ \beta^{m-m_1-1} T_0\left(\underline{x},\beta\right) \int_\theta \frac{\theta^{m+\alpha-1}}{e^{\theta(T_1(\underline{x})+T_2(\underline{x},\beta)+1)}} \int_\sigma \frac{\sigma^{m+\gamma-1}}{e^{\sigma-T_3(\underline{x})-T_4(\underline{x},\beta)}}\, d\sigma\, d\theta \tag{21}$$

and

$$\pi^*_\sigma = \Omega\ \frac{\sigma^{m+\gamma-1}}{e^{\sigma}} \int_\theta \frac{\theta^{m+\alpha-1}}{e^{\theta(T_1(\underline{x})+1)}\ e^{-T_3(\underline{x})}} \int_\beta \beta^{m-m_1-1} \frac{T_0\left(\underline{x},\beta\right)}{e^{\theta T_2(\underline{x},\beta)-T_4(\underline{x},\beta)}}\, d\beta\, d\theta \tag{22}$$

where $\Omega = \left\{\int_\theta \theta^{m+\alpha-1} e^{-\theta T_1(\underline{x})+1} \int_\sigma \frac{\sigma^{m+\gamma-1}}{e^{\sigma-T_3(\underline{x})}} \int_\beta \beta^{m-m_1-1} \frac{T_0(\underline{x},\beta)}{e^{\theta T_2(\underline{x},\beta)-T_4(\underline{x},\beta)}}\, d\beta\, d\sigma\, d\theta\right\}^{-1}$.

The Bayes predictive density $h_\Theta\left(y|\underline{x}\right)\ ;\ \Theta\left(=\theta,\sigma,\beta\right)$, for future observation Y is defined and obtained as

$$h_\Theta\left(y|\underline{x}\right) \propto \int_\Theta f\left(y;\sigma,\theta\right) \times \pi^*_\Theta\, d\Theta\ ;\ \Theta = \theta,\sigma,\beta \tag{23}$$

Using the Bayes predictive density $h_\Theta\left(y|\underline{x}\right)\ ;\ \Theta\left(=\theta,\sigma,\beta\right)$, for the future observation Y, the cumulative density function is obtained as

$$G_\Theta\left(y|\underline{x}\right) = Pr\left(Y \le y\right) = \int_0^y h_\Theta\left(y|\underline{x}\right) dy$$

$$\Rightarrow G_\Theta\left(y|\underline{x}\right) \propto \int_\Theta \left\{1-\left(1-e^{-\frac{\theta}{y}}\right)^\sigma\right\} \times \pi^*_\Theta\, d\Theta\ ;\ \Theta = \theta,\sigma,\beta. \tag{24}$$

If Y_j denotes the j^{th} order statistic in future sample $m\left(1 \le j \le m\right)$, then the probability density function is defined for the j^{th} order statistic as

$$\phi_\Theta\left(Y_j\right) = j\left({}^mC_j\right)\left(G_\Theta\left(y|\underline{x}\right)\right)^{j-1}\left(1-G_\Theta\left(y|\underline{x}\right)\right)^{m-j} h_\Theta\left(y|\underline{x}\right)\ ;\ \Theta = \theta,\sigma,\beta. \tag{25}$$

The lower and upper Bayes prediction bound limits corresponding to the parameter $\Theta\left(=\theta,\sigma,\beta\right)$ for the j^{th} item are obtained by solving following equalities

$$j\left({}^mC_j\right)\int_0^{l_1} Z^{j-1}\left(1-Z\right)^{m-j}\ dZ = \frac{\tau}{2} \tag{26}$$

and

$$j\left({}^mC_j\right)\int_0^{l_2} Z^{j-1}\left(1-Z\right)^{m-j}\ dZ = 1-\frac{\tau}{2}, \tag{27}$$

where $l_i = \int_\Theta \left\{1-\left(1-e^{-\frac{\theta}{l_{ij}}}\right)^\sigma\right\} \times \pi^*_\Theta\, d\Theta\ ;\ \Theta = \theta,\sigma,\beta$ and $i = 1,2$.

The Eq. (26) & Eq. (27), will be solved under the given limits for the smallest future observation $(j = 1)$ and for the largest future observation $(j = m)$. The explicit expressions of TS-BPBL does not exists corresponding to the parameters θ, σ and β. If l_{21} and l_{11} are the upper and lower limits for the smallest future observations, then the TS-BPBL for the smallest future observation is obtained from

$$I_S = l_{21} - l_{11}. \tag{28}$$

Similarly, if l_{2m} and l_{1m} are the upper and lower limits for the largest future observation, then the TS-BPBL for the largest future observation (e.g.,$j = m$) is obtained from

$$I_L = l_{2m} - l_{1m}. \tag{29}$$

5. Numerical analysis on simulated data

A simulation study has been performed in the present section for the analysis of the proposed methods. The Monte Carlo simulation technique was used for generating 10,000 FFP censored samples for each simulation by using algorithms described by Balakrishnan and Sandhu (1995).

The samples were simulated for $n = 30, m = 10, 15$ with hyper-parametric values $\alpha = \gamma\,(= 0.25, 0.75, 1.25, 2.00, 5.00)$ and different values of k given in Table 1. All the special cases of FFPC have considered in this section for the analysis.

Table 1: Special cases of FFP censoring scheme

Case	k	m	$R_i; 1, 2, ...m,$	Different Censoring Plans
1	5	10	1 2 0 2 1 1 0 0 5 0	First-Failure Progressive Type-II Censoring (FFPC)
2	1	10	1 2 0 2 1 1 0 0 5 0	Progressive Type-II Censoring (PC)
3	5	10	0 0 0 0 0 0 0 0 0 0	First-Failure Censoring (FFC)
4	1	10	0 0 0 0 0 0 0 0 0 20	Type-II Censoring (T-II)
5	1	10	0 0 0 0 0 0 0 0 0 0	Complete Sample (CS)
1	5	15	1 2 0 2 1 1 0 0 5 3 0 1 4 2 1	First-Failure Progressive Type-II Censoring (FFPC)
2	1	15	1 2 0 2 1 1 0 0 5 0 0 1 4 2 1	Progressive Type-II Censoring (PC)
3	5	15	0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	First-Failure Censoring (FFC)
4	1	15	0 0 0 0 0 0 0 0 0 0 0 0 0 0 15	Type-II Censoring (T-II)
5	1	15	0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	Complete Sample (CS)

The values of the parameters under consideration were assumed here as $(\theta, \sigma) = (2.01, 0.39)$, $(3.50, 3.67)$, $(5.02, 6.98)$. The selection of these values meets the criterion that the variance should be unity. The optimal change stress time ϵ is calculated by the method of minimizing the asymptotic variance of ML Estimation of the parameter θ and the acceleration factor β. The asymptotic variances of these parameters are calculated from the diagonal elements of the inverse of Fisher information matrix given in Eq. (13). The optimal change stress time is the value, which minimizes asymptotic variance of ML Estimate and, obtained by using Wolfram Mathematica software 10.0.

Table 2, presents the Maximum likelihood estimate of the parameter θ and the acceleration factor β. The assumed values of the acceleration factor for the numerical findings are $\beta(= 0.50(0.10)2.50)$ and it is observed that the ML estimate, first increases as β increases and reaches maximum when $\beta = 1.40$ and then decrease, however the magnitude is nominal for both the parameters. Hence, the results are shown here only for the $\beta = 1.40$.

It is further noted that, the maximum magnitude was noted for FFP censoring scheme over the other one, whereas the complete sample case shows second maximum estimate as compared to other censoring schemes. It is also observed that, as the parameter (θ, σ) increases,

Table 2: Maximum likelihood estimate

$\beta = 1.40$	$\leftarrow\ \theta, \sigma\ \rightarrow$					
	$m_1 = 05, m - m_1 = 05$			$m_1 = 10, m - m_1 = 05$		
$\hat{\theta}_{ML}$	2.01, 0.39	3.50, 3.67	5.02, 6.98	2.01, 0.39	3.50, 3.67	5.02, 6.98
FFPC	0.9201	0.9521	0.9829	1.0453	1.0507	1.0674
PC	0.7803	0.7877	0.7948	0.8852	0.8936	0.9291
FFC	0.6798	0.6912	0.6966	0.7765	0.7831	0.8101
T-II	0.8516	0.8561	0.8642	0.9621	0.9716	1.0132
CS	0.9024	0.9103	0.9457	1.0228	1.0283	1.0375
	$\leftarrow\ \theta, \sigma\ \rightarrow$					
$\hat{\beta}_{ML}$	2.01, 0.39	3.50, 3.67	5.02, 6.98	2.01, 0.39	3.50, 3.67	5.02, 6.98
FFPC	0.9409	0.9157	0.9175	1.0646	1.0131	1.0009
PC	0.8029	0.7534	0.7318	0.9065	0.8579	0.8644
FFC	0.7037	0.6581	0.6348	0.7992	0.7488	0.7468
T-II	0.8733	0.8209	0.8003	0.9823	0.9350	0.9474
CS	0.9235	0.8744	0.8808	1.0424	0.9910	0.9714

the ML estimate of the parameter θ was increased, however the ML estimate of σ has not been shown similar tendency. Also, as the censored sample size increases the estimate of the parameters increases.

Table 3, presents the approximate confidence lengths for the parameter θ and the acceleration factor β under the normal approximation, given in Eq. (14) and Eq. (15). It is observed that the length increases as the confidence level or the censoring size increases. The maximum lengths, noted in the FFP censoring pattern, whereas the minimum length were observed in the complete sample case. The increasing trend in length was also seen first when parameter β increases up to $(\beta = 1.40)$, and then decreases.

Table 4 - 6 presents the Two-Sample Bayes prediction bound length (TS-BPBL) for the parameters θ, σ and β respectively for the assumed parametric values. The maximum bound lengths have been noted in the prior parametric value $\alpha = 1.25 = \gamma$ and parametric values $(\theta, \sigma) = (3.50, 3.67)$. Other properties have been seen similar as discussed above. Hence, based on above parametric values, it is observed that the FFP censoring scheme provides wider length as compared to progressing or conventional censoring pattern.

6. Numerical analysis on real data

In this section, we consider $n = 23$ deep-groove ball bearing failure times for the numerical analysis in present discussion, originally used by Lieblein and Zelen (1956). The said data set represents the number of millions of revolutions before failure for each of the 23 ball bearings in a life test and the data's are 17.88, 28.92, 33.0, 41.52, 42.12, 45.60, 48.40, 51.84, 51.96, 54.12, 55.56, 67.80, 68.64, 68.64, 68.88, 84.12, 93.12, 98.64, 105.12, 105.84, 127.92, 128.04, 173.40.

Based on above assumed parametric values, the numerical findings were presented in the Tables 7 - 9. The similar increasing trend have noted for the parameter β up to $(\beta = 1.40)$. Hence, all the results have presents here only for $(\beta = 1.40)$.

All the properties have seen similar as discussed above. However, the magnitude of the ML estimate were smaller whereas the TS-BPBL become wider. This shows that the above approach presents good numerical finding under the real data set.

Table 3: Approximate confidence length

$\leftarrow \theta \rightarrow$		$m_1 = 05, m - m_1 = 05$			$m_1 = 10, m - m_1 = 05$		
$\tau \rightarrow$	$\beta \downarrow$	90%	95%	99%	90%	95%	99%
FFPC	0.50	0.6641	0.6767	0.6929	0.7477	0.7753	0.7852
	1.40	0.7611	0.8201	0.8219	0.8894	0.9023	0.9119
	2.50	0.7239	0.7718	0.8106	0.8244	0.8668	0.8783
PC	0.50	0.6509	0.6579	0.6891	0.7331	0.7515	0.7808
	1.40	0.7584	0.8092	0.8126	0.8851	0.9001	0.9097
	2.50	0.7161	0.7367	0.7634	0.8064	0.8369	0.8543
FFC	0.50	0.5777	0.6002	0.6153	0.6507	0.6775	0.6798
	1.40	0.6927	0.7283	0.7358	0.7801	0.7909	0.8086
	2.50	0.6179	0.6457	0.6482	0.6961	0.7248	0.7674
T-II	0.50	0.6361	0.6571	0.6659	0.7163	0.7441	0.7477
	1.40	0.7459	0.7636	0.7918	0.8397	0.8605	0.8701
	2.50	0.6727	0.7013	0.7053	0.7576	0.7874	0.8348
CS	0.50	0.5053	0.5159	0.5299	0.5692	0.5951	0.5953
	1.40	0.6061	0.6282	0.6443	0.6826	0.6971	0.7082
	2.50	0.5406	0.5669	0.5676	0.6091	0.6363	0.6721
$\leftarrow \beta \rightarrow$		$m_1 = 05, m - m_1 = 05$			$m_1 = 10, m - m_1 = 05$		
FFPC	0.50	0.6736	0.6921	0.7047	0.7588	0.7926	0.7988
	1.40	0.7724	0.8182	0.8362	0.9032	0.9220	0.9279
	2.50	0.7345	0.7890	0.8247	0.8369	0.8858	0.8937
PC	0.50	0.6602	0.6730	0.7008	0.7438	0.7684	0.7943
	1.40	0.7697	0.8172	0.8267	0.8987	0.9198	0.9257
	2.50	0.7266	0.7533	0.7766	0.8186	0.8554	0.8692
FFC	0.50	0.5856	0.6142	0.6257	0.6599	0.6793	0.6914
	1.40	0.7027	0.7447	0.7485	0.7917	0.8085	0.8226
	2.50	0.6265	0.6306	0.6592	0.7061	0.7412	0.7807
T-II	0.50	0.6451	0.6721	0.6772	0.7268	0.7307	0.7606
	1.40	0.7570	0.7807	0.8055	0.8525	0.8794	0.8853
	2.50	0.6824	0.7172	0.7174	0.7689	0.8049	0.8493
CS	0.50	0.5118	0.5283	0.5387	0.5769	0.5969	0.6053
	1.40	0.6144	0.6331	0.6552	0.6925	0.7171	0.7203
	2.50	0.5378	0.5503	0.5771	0.6175	0.6510	0.6835

7. Conclusion

The generalized Inverted Exponential distribution is taken here for the study on Optimum Step Stress Partially Accelerated Life Test (SS-PALT) based on different special censoring patterns of the first-failure progressive censoring (FFPC). The maximum likelihood estimate, approximate confidence limit under the normal approximation and two-sample Bayes Prediction Bound Length (TS-BPBL) have been obtained and studied their properties by using simulated and real data set. One may preferred the FFP censoring pattern over the progressing or conventional censoring based on above selected parametric values.

Table 4: Two-sample Bayes prediction bound length (for parameter θ)

$j=1$		$m_1=05, m-m_1=05$			$m_1=10, m-m_1=05$		
$\alpha=1.25=\gamma,\ (\theta,\sigma)=(3.50,3.67)$							
$\tau\rightarrow$	$\beta\downarrow$	90%	95%	99%	90%	95%	99%
FFPC	0.50	0.7467	0.7704	0.7748	0.8399	0.8690	0.8776
	1.40	0.8134	0.8540	0.9059	0.9253	0.9598	0.9813
	2.50	0.8548	0.9077	0.9185	0.9977	1.0105	1.0188
PC	0.50	0.7320	0.7605	0.7705	0.8235	0.8648	0.8727
	1.40	0.8047	0.8483	0.8533	0.9052	0.9488	0.9546
	2.50	0.8518	0.8957	0.9082	0.9928	1.0070	1.0163
FFC	0.50	0.6505	0.6740	0.6772	0.7318	0.7490	0.7602
	1.40	0.6953	0.7135	0.7250	0.7823	0.8017	0.8578
	2.50	0.7786	0.8167	0.8226	0.8758	0.8965	0.9037
T-II	0.50	0.7155	0.7406	0.7447	0.8049	0.8230	0.8359
	1.40	0.7563	0.7755	0.7886	0.8509	0.8714	0.9329
	2.50	0.8378	0.8783	0.8850	0.9424	0.9640	0.9722
CS	0.50	0.5698	0.5912	0.5932	0.6410	0.6570	0.6661
	1.40	0.6091	0.6258	0.6352	0.6853	0.7031	0.7515
	2.50	0.6820	0.7163	0.7207	0.7673	0.7863	0.7918
$j=m$		$m_1=05, m-m_1=05$			$m_1=10, m-m_1=05$		
$\tau\rightarrow$	$\beta\downarrow$	90%	95%	99%	90%	95%	99%
FFPC	0.50	0.9030	0.9286	0.9381	1.0161	1.0582	1.0629
	1.40	0.9840	1.0401	1.0972	1.1198	1.1684	1.1887
	2.50	1.0342	1.1052	1.1125	1.2076	1.2299	1.2342
PC	0.50	0.8852	0.9266	0.9329	0.9962	1.0531	1.0569
	1.40	0.9734	1.0331	1.0334	1.0954	1.1551	1.1563
	2.50	1.0306	1.0906	1.1012	1.2017	1.2257	1.2312
FFC	0.50	0.7863	0.7926	0.8197	0.8849	0.9126	0.9204
	1.40	0.8406	0.8695	0.8777	0.9462	0.9766	1.0388
	2.50	0.9417	0.9948	0.9961	1.0597	1.0916	1.0945
T-II	0.50	0.8652	0.8924	0.9016	0.9736	1.0024	1.0123
	1.40	0.9147	0.9448	0.9549	1.0295	1.0611	1.1301
	2.50	1.0136	1.0695	1.0718	1.1405	1.1735	1.1777
CS	0.50	0.6884	0.7111	0.7277	0.7748	0.8010	0.8062
	1.40	0.7360	0.7631	0.7687	0.8285	0.8569	0.9098
	2.50	0.8245	0.8429	0.8725	0.9280	0.9579	0.9587

Table 5: Two-sample Bayes prediction bound length (for parameter σ)

$j = 1$		$m_1 = 05, m - m_1 = 05$			$m_1 = 10, m - m_1 = 05$		
$\alpha = 1.25 = \gamma,\ (\theta, \sigma) = (3.50, 3.67)$							
$\tau \rightarrow$	$\beta \downarrow$	90%	95%	99%	90%	95%	99%
FFPC	0.50	0.7251	0.7501	0.7537	0.8161	0.8504	0.8541
	1.40	0.7902	0.8358	0.8818	0.8995	0.9391	0.9554
	2.50	0.8307	0.8882	0.8941	0.9702	0.9886	0.9920
PC	0.50	0.7107	0.7445	0.7495	0.8001	0.8463	0.8494
	1.40	0.7817	0.8302	0.8304	0.8799	0.9284	0.9293
	2.50	0.8277	0.8765	0.8840	0.9654	0.9852	0.9896
FFC	0.50	0.6312	0.6461	0.6584	0.7106	0.7332	0.7395
	1.40	0.6749	0.6986	0.7051	0.7599	0.7847	0.8348
	2.50	0.7563	0.7994	0.8004	0.8512	0.8773	0.8796
T-II	0.50	0.6946	0.7125	0.7243	0.7819	0.8055	0.8134
	1.40	0.7345	0.7591	0.7672	0.8269	0.8528	0.9081
	2.50	0.8141	0.8595	0.8614	0.9162	0.9432	0.9465
CS	0.50	0.5523	0.5711	0.5764	0.6219	0.6434	0.6476
	1.40	0.5907	0.6129	0.6174	0.6651	0.6884	0.7310
	2.50	0.6619	0.6803	0.7009	0.7452	0.7697	0.7703
$j = m$		$m_1 = 05, m - m_1 = 05$			$m_1 = 10, m - m_1 = 05$		
$\tau \rightarrow$	$\beta \downarrow$	90%	95%	99%	90%	95%	99%
FFPC	0.50	0.8845	0.9136	0.9206	0.9960	1.0264	1.0436
	1.40	0.9643	1.0285	1.0775	1.0982	1.1551	1.1677
	2.50	1.0139	1.0627	1.0926	1.1848	1.1957	1.2125
PC	0.50	0.8669	0.9067	0.9154	0.9764	1.0114	1.0378
	1.40	0.9539	1.0017	1.0145	1.0742	1.0942	1.1357
	2.50	1.0102	1.0584	1.0802	1.1789	1.1816	1.2095
FFC	0.50	0.7695	0.7962	0.8038	0.8668	0.8929	0.9032
	1.40	0.8230	0.8605	0.8610	0.9272	0.9659	1.0199
	2.50	0.9228	0.9584	0.9778	1.0390	1.0494	1.0748
T-II	0.50	0.8472	0.8775	0.8846	0.9541	0.9914	0.9937
	1.40	0.8961	0.9346	0.9371	1.0092	1.0494	1.1097
	2.50	0.9936	1.0276	1.0525	1.1186	1.1401	1.1568
CS	0.50	0.6729	0.6943	0.7034	0.7581	0.7729	0.7906
	1.40	0.7199	0.7355	0.7536	0.8110	0.8348	0.8928
	2.50	0.8071	0.8381	0.8559	0.9092	0.9176	0.9409

Table 6: Two-sample Bayes prediction bound length (for acceleration factor β)

$j=1$		$m_1=05, m-m_1=05$			$m_1=10, m-m_1=05$		
$\alpha=1.25=\gamma$, $(\theta,\sigma)=(3.50,3.67)$							
$\tau \rightarrow$	$\beta \downarrow$	90%	95%	99%	90%	95%	99%
FFPC	0.50	0.8159	0.8473	0.8545	0.9188	0.9562	0.9681
	1.40	0.8896	0.9396	0.9993	1.0132	1.0565	1.0826
	2.50	0.9353	0.9989	1.0132	1.0931	1.1125	1.1240
PC	0.50	0.7996	0.8363	0.8498	0.9007	0.9515	0.9627
	1.40	0.8799	0.9333	0.9412	0.9910	1.0443	1.0531
	2.50	0.9320	0.9857	1.0019	1.0877	1.1086	1.1213
FFC	0.50	0.7096	0.7408	0.7467	0.7994	0.8236	0.8384
	1.40	0.7591	0.7844	0.7995	0.8552	0.8818	0.9462
	2.50	0.8511	0.8984	0.9073	0.9585	0.9865	0.9969
T-II	0.50	0.7814	0.8143	0.8213	0.8802	0.9054	0.9220
	1.40	0.8265	0.8529	0.8698	0.9310	0.9588	1.0292
	2.50	0.9165	0.9664	0.9762	1.0320	1.0611	1.0726
CS	0.50	0.6205	0.6493	0.6539	0.6991	0.7220	0.7345
	1.40	0.6639	0.6875	0.7003	0.7481	0.7729	0.8288
	2.50	0.7444	0.7875	0.7948	0.8386	0.8648	0.8733
$j=m$		$m_1=05, m-m_1=05$			$m_1=10, m-m_1=05$		
$\tau \rightarrow$	$\beta \downarrow$	90%	95%	99%	90%	95%	99%
FFPC	0.50	0.9717	1.0170	1.0188	1.0947	1.1471	1.1546
	1.40	1.0598	1.1273	1.1919	1.2075	1.2670	1.2914
	2.50	1.1144	1.1982	1.2085	1.3029	1.3339	1.3409
PC	0.50	0.9522	1.0039	1.0132	1.0730	1.1415	1.1481
	1.40	1.0482	1.1198	1.1224	1.1809	1.2524	1.2561
	2.50	1.1104	1.1824	1.1950	1.2965	1.3293	1.3376
FFC	0.50	0.8447	0.8897	0.8900	0.9520	0.9887	0.9996
	1.40	0.9038	0.9418	0.9531	1.0187	1.0582	1.1284
	2.50	1.0138	1.0781	1.0819	1.1421	1.1834	1.1890
T-II	0.50	0.9305	0.9776	0.9791	1.0485	1.0864	1.0995
	1.40	0.9844	1.0237	1.0371	1.1092	1.1503	1.2276
	2.50	1.0919	1.1593	1.1642	1.2299	1.2725	1.2794
CS	0.50	0.7382	0.7504	0.7791	0.8321	0.8673	0.8754
	1.40	0.7901	0.8261	0.8345	0.8907	0.9281	0.9881
	2.50	0.8862	0.9456	0.9475	0.9988	1.0379	1.0413

Table 7: Approximate confidence length (based on real data)

		$m_1=05, m-m_1=05$			$m_1=10, m-m_1=05$		
	$\tau \rightarrow$	90%	95%	99%	90%	95%	99%
θ	FFPC	0.7654	0.8306	0.8486	0.8951	0.9137	0.9195
	PC	0.7627	0.8197	0.8292	0.8905	0.9115	0.9173
	FFC	0.6964	0.7381	0.7417	0.7845	0.8012	0.8152
	T-II	0.7501	0.7737	0.7982	0.8448	0.8715	0.8773
	CS	0.6088	0.6369	0.6493	0.6862	0.7065	0.7138
β	FFPC	0.7806	0.8472	0.8689	0.9128	0.9317	0.9377
	PC	0.7778	0.8358	0.8594	0.9082	0.9295	0.9355
	FFC	0.7102	0.7525	0.7564	0.8001	0.8173	0.8313
	T-II	0.7651	0.7889	0.8141	0.8616	0.8887	0.8947
	CS	0.6209	0.6494	0.6622	0.6998	0.7204	0.7279

Table 8: TS-BPBL based on real data for $j = 1$

$(\theta,\sigma) = (3.50, 3.67)$		$m_1 = 05, m - m_1 = 05$			$m_1 = 10, m - m_1 = 05$		
$\alpha = 1.25 = \gamma$	$\tau \rightarrow$	90%	95%	99%	90%	95%	99%
θ	FFPC	0.8182	0.8649	0.9134	0.9312	0.9718	0.9896
	PC	0.8095	0.8592	0.8603	0.9109	0.9607	0.9626
	FFC	0.6991	0.7231	0.7308	0.7868	0.8121	0.8649
	T-II	0.7606	0.7857	0.7952	0.8561	0.8825	0.9407
	CS	0.6119	0.6345	0.6401	0.6889	0.7126	0.7575
β	FFPC	0.8937	0.9441	1.0044	1.0184	1.0621	1.0884
	PC	0.8839	0.9378	0.9457	0.9964	1.0498	1.0586
	FFC	0.7621	0.7876	0.8028	0.8591	0.8858	0.9508
	T-II	0.8301	0.8567	0.8737	0.9355	0.9635	1.0345
	CS	0.6661	0.6898	0.7027	0.7517	0.7767	0.8324
σ	FFPC	0.8173	0.8647	0.9124	0.9308	0.9719	0.9888
	PC	0.8085	0.8589	0.8591	0.9104	0.9608	0.9617
	FFC	0.6977	0.7223	0.7294	0.7859	0.8116	0.8636
	T-II	0.7595	0.7851	0.7935	0.8554	0.8823	0.9397
	CS	0.6103	0.6333	0.6382	0.6875	0.7117	0.7559

Table 9: TS-BPBL based on real data for $j = m$

$(\theta,\sigma) = (3.50, 3.67)$		$m_1 = 05, m - m_1 = 05$			$m_1 = 10, m - m_1 = 05$		
$\alpha = 1.25 = \gamma$	$\tau \rightarrow$	90%	95%	99%	90%	95%	99%
θ	FFPC	0.9905	1.0529	1.1066	1.1276	1.1824	1.1999
	PC	0.9798	1.0358	1.0422	1.1035	1.1591	1.1663
	FFC	0.8457	0.8806	0.8857	0.9523	0.9887	1.0476
	T-II	0.9205	0.9566	0.9629	1.0365	1.0741	1.1398
	CS	0.7401	0.7732	0.7749	0.8335	0.8679	0.9174
β	FFPC	1.0633	1.1314	1.1965	1.2122	1.2721	1.2967
	PC	1.0516	1.1238	1.1264	1.1854	1.2574	1.2612
	FFC	0.9061	0.9444	0.9558	1.0219	1.0617	1.1325
	T-II	0.9873	1.0269	1.0405	1.1131	1.1545	1.2324
	CS	0.7915	0.8278	0.8363	0.8929	0.9306	0.9911
σ	FFPC	0.9836	1.0499	1.1005	1.1219	1.1806	1.1937
	PC	0.9729	1.0222	1.0354	1.0971	1.1178	1.1606
	FFC	0.8377	0.8764	0.8769	0.9453	0.9852	1.0413
	T-II	0.9132	0.9529	0.9555	1.0309	1.0715	1.1338
	CS	0.7312	0.7473	0.7661	0.8253	0.8499	0.9098

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